Identifying and Estimating Beliefs from Choice Data - An Application to Female Labor Supply

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February 19, 2020

Abstract

This paper investigates the life cycle costs of biased beliefs of future employment possibilities, focusing on females who experience child-related career breaks. To estimate these costs, I develop a novel strategy to identify expectations of employment prospects within a life cycle model of female labor supply and human capital accumulation. Reactions to a discontinuity in the future expected value of non-employment caused by the end of employment protection allows for identification of expectations. In addition, reforms that exogenously vary the length of this protection period allow to separately identify expectations, job-arrival rates, and preferences individually. In line with suggestive evidence, the estimated life cycle model indicates that expectations are substantially biased: on average women expect the half-yearly job arrival rate to be twice the actual rate. This overconfidence prolongs the average child related career break by eight months, resulting in a larger share of mothers staying non-employed beyond the protection period. The implications of forgone wages and human capital are large, since overconfidence decreases life-time earnings from employment by 14%.

JEL: D84, J24, H30

Keywords: biased beliefs, labor supply, dynamic discrete choice, identification, maternity leave

Acknowledgements: I gratefully acknowledge financial support from German Research Foundation (DFG) grants CRC TRR-190 and SPP-1764. I thank Peter Haan, Fedor Iskhakov, Georg Weizsäcker, Richard Blundell, Thierry Magnac, Andreas Haufler, and Ludger Wößmann for helpful feedback. I also would like to thank the HPC Service of ZEDAT, Freie Universität Berlin, for computing time.

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1 Introduction

It is well known that beliefs are often systematically biased. Drivers have upwards biased beliefs of how safe they drive, students have biased expectations of their humor, grammar, and logic skills with respect to others, while finance professionals overestimate the precision of their stock market predictions.[1] In the labor market, individuals might have biased expectations of their future employment prospects. Career costs caused by overestimating future employment prospects can be especially high for mothers who interrupt their working careers after giving birth to children. If mothers are too optimistic about their employment prospects, they might not return during maternity leave, a period when their return to their previous job is guaranteed, although optimal under unbiased expectations. After maternity leave, mothers have to rely on new employment offers that arrive with a lower probability as anticipated. Therefore, overestimating future employment prospects prolongs child-related employment interruptions, increasing the career costs of having children.[2] In contrast, if mothers underestimate their future employment opportunities on average, more women return to employment during maternity leave than under rational expectations, reducing the costs of motherhood. Previous literature acknowledges the importance of potentially biased expectations, but does not identify these within the context of child-related career breaks or quantifies their consequences for the working careers of mothers.

In this paper, I develop a life cycle model of female labor supply and human capital accumulation, derive a strategy to identify job offer expectations within this model, and quantify the career costs of biased expectations of future employment prospects. To estimate the model, I use survey data from the German Socio-Economic Panel Study (SOEP), since German maternity leave regulations provide an ideal environment to identify the key parameters of the model. The identification approach exploits the impact of expectations on the decision process at the end of maternity leave, which provides mothers with a guarantee to return to their previous job. This change from an employment guarantee to a situation in which individuals have to rely on job offers to leave non-employment creates a discontinuity in the future expected value of non-employment that varies

[1] See Svenson (1981), Kruger & Dunning (1999) and Ben-David et al. (2013). (De Bondt & Thaler, 1995, p. 389) even conclude, “perhaps the most robust finding in the psychology of judgment is that people are overconfident.”

[2] Children are one important factor for the career dynamics of women. This is reflected by the average employment rates across OECD countries, which are 11 percent lower for women with at least one child (aged 0-14) than for women without a child in 2014. The career costs of having children can be high, for instance, Adda et al. (2017) estimate that fertility reduces the net present value of income by 35%, of which they attribute 76% to the lower employment of mothers.

with job offer expectations. To separately identify expectations, preferences, and real job offer rates, several maternity leave reforms are exploited, which change the length of the employment protection.

Expectations of future employment opportunities are modeled, such that rational expectations are nested, which is the predominant assumption when estimating life cycle models of female labor supply. This allows for directly testing if beliefs about future employment opportunities are biased. In a second step, I quantify the life cycle costs of these biased expectations. Holding the preference parameters constant, but restricting expectations to be rational, I simulate life cycle choices and compare them with actual observed choices. I can, thus, examine the welfare costs of wrongly estimating future employment possibilities.

In the first part of the paper, I develop a life-cycle model of female labor supply and human capital accumulation (see for example Keane et al., 2011).[3] In this model, women choose their labor supply in half-yearly intervals, facing labor market frictions. To enter employment after being non-employed, women need to receive a job offer, which arrives with a probability related to their age and human capital. In contrast to the standard life-cycle framework, which assumes that individuals have rational expectations, I explicitly model expectations of the future job offer arrival probability. These expectations are especially crucial when deciding whether or not to return within the employment-protected maternity leave period. If an individual overestimates her future employment opportunities, she might not return during the employment protection. Having overestimated her chances to receive a job offer, the career break is, on average, longer than she expected. Because on-the-job human capital depreciates when non-employed, longer non-employment spells are not only more costly because of the lost income from employment, but also because of the losses in human capital.

In the second part of the paper, I present a novel approach to identify the job offer expectations within the discussed life-cycle model using only choice data. The approach relies on the impact of expectations on the decision process of returning to employment at the end of maternal leave. If mothers expect the arrival rate of future job offers to be very high, the future expected cost of not returning to their protected job are low. If, in contrast, the expectations of the future job offer

[3] Research employing a life-cycle model of female labor supply include, for example, Blundell et al. (2016) who focus on how welfare reforms within a life-cycle model of labor supply and human capital, Adda et al. (2017) who evaluate how fertility influences occupational and employment choices over the life-cycle, and Low et al. (2010) who investigate the influence of different types of risks that individuals face over their working life-cycle.

probability are rather low, the future expected costs, in terms of lost income and human capital, are relatively high. Essentially, the lower the expected job offer rate, the higher the expected costs of not returning to the guaranteed job and the higher the probability of women returning at the end of the employment protection. Therefore, the mass of mothers returning directly at the end of the employment protection is at least partly a result of the mothers' expectations of their future employment opportunities.[4]

Although expectations influence the returning behavior of mothers at the end of their employment protection period, there are other factors that might drive returning behavior. To control for other potential influences, I exploit several reforms of the German maternity leave regulations. These reforms first extend the employment protection period from 1 year to 1.5 years, and then to 3 years. The three policy regimes create three groups of individuals facing different lengths of employment protection when the youngest child is between 1 and 1.5 years old. Employment rates of mothers in the regime with the longest lasting employment protection aid the identification of leisure preferences regarding the age of the youngest child. Their returns to employment around 1.5 years after childbirth are not influenced by the expectations of future employment possibilities, but are solely driven by leisure preferences. Comparing the mass of returning mothers shortly before the youngest child turns 1.5 between the regime that grants mothers 3 years of employment protection and the regime that grants mothers 1.5 years, identifies the excess mass due to expectations of future job offers. Having identified preferences and expectations, it is possible to use the non-employment to employment transitions of mothers with children older than 1.0 of the regime with the shortest employment protection to identify the real job offer rate.

In the third part of the paper, I estimate the model and quantify the career costs of overconfidence. Individuals receive full-time employment offers with around 50% in a year, and part-time employment offers with around 17%, but these probabilities decrease with the time spend in non-employment. Individuals display a strong bias in their expectations, and anticipate the job arrival rate to be 66% higher compared to the real rates, on average. These findings are in line with the

[4]The identification approach has some similarity to the literature using bunching for identification of elasticities (see Saez (2010) and Kleven & Waseem (2013)). The classical bunching approach would use a kink or notch in the tax schedule to recover underlying labor supply elasticities. In contrast, this paper uses a discontinuity over time in the guarantee of returning to a previous job. Additionally, the counterfactual situation of not having this discontinuity is available, since policy reforms prolonged the length of employment protection over the years. Another paper using bunching to identify welfare costs of behavioral elements instead of elasticities is Rees-Jones (2018). He tries to quantify the tax evasion costs introduced by loss-aversion when individuals owes taxes at the end of a tax year instead of receiving a refund.

suggestive evidence constructed from several questions of the SOEP questionnaire about future employment expectations. Simulating the model once with the estimated expectations and once with rational expectations allows for quantifying the costs of overconfidence. Under biased expectations, child related career breaks are, on average, 6 months longer. Women lose between 12% and 18% of the net present value of earnings from employment. The net present value in household consumption is much lower, lying between 3% and 4%. There are two main reasons for this difference. First, partners contribute the larger share to the overall household income, since they are mostly working full-time (and do not interrupt their career due to childbirth), while mothers re-entering the labor market typically work part-time. Second, the German tax system, with its joint taxation system, heavily taxes second earner income. The simulations also show that the costs of overconfidence decrease with the length of the employment protection.

The life-cycle loss in earnings from employment due to biased expectations are meaningful from a public economics perspective. They resemble losses in income taxation in addition to the possible social security provided to mothers who have not returned to employment due to their biased expectations. In addition, the consequences for the individual are substantial: The lost lifetime earnings translate into lower pension benefits making them more vulnerable to poverty in retirement. The consequences might justify interventions by policy makers. Potential policies might provide more information about employment prospects after child-related career breaks, for instances by introducing mandatory consulting meetings with an employment agency. Other measures might include financially incentivizing returning to work within the employment protection, for example by providing in-work benefits toward the end of the employment protection.

Contribution to the Literature

The two major contributions of this paper are, first, identifying beliefs from actual employment behavior over time and, second, estimating the life-cycle costs of biased beliefs. These two contributions connect the behavioral literature and the literature on life-cycle labor supply. I extend the behavioral literature, which predominantly derives its empirical evidence from specially designed experiments, by finding evidence of biased beliefs in the context of labor supply choices over time. Furthermore, I can estimate the long-term consequences of the bias, which is not often possible in an experimental setting. The contribution to the literature on life-cycle employment behavior comes from allowing non-rational expectations. This is often ignored because an identification strategy is not available.

In general, there is little evidence of biased beliefs and its consequences using a revealed preference approach outside of laboratory experiments. Since the work of Tversky & Kahneman (1974), which introduces a theory for their common finding that individuals exhibit systematic biases when acting under uncertainty, the literature on social psychology and organizational behavior[5] intensively analyzes overconfidence. An introduction into the literature's link to economic questions is provided by Malmendier & Taylor (2015).[6] The majority of the findings stem from experiments, since most surveys capture expectations too broadly to provide convincing evidence on overconfidence. Although laboratory experiments are ideal for exploring behavior and testing possible theories about the decision making process, these might not be well suited to quantify real economic consequences. This paper closes this gap by identifying expectations from observed choice data within a life-cycle model of labor supply.

The labor economics literature investigating expectations outside experiments can be divided into two parts.[7] One part uses subjective data in reduced form analysis to determine the impact of expectations on labor outcomes. Most of this research investigates how future earnings and labor market attachment expectations influence education and other investment in human capital decisions. For example, Sandell & Shapiro (1980) and Shaw & Shapiro (1987) show that individuals who do not expect strong future labor attachment, invest less in human capital than individuals with stronger expected attachment. This is further underlined by Gronau (1988) and Blau & Ferber (1991). The other part of the literature concentrates on testing more directly if expectations are unbiased by comparing surveyed expectations with actual behavior. For instance, Hamermesh (1985), Bernheim (1988), and Hurd et al. (2004) find individuals are mostly able to predict their retirement age.

[5] Moore & Healy (2008) survey this literature.

[6] For seminal work, see, for example, Svenson (1981) who finds that 83% of participants in a laboratory experiment stated that they are in the top 30% regarding driving safety, Kruger & Dunning (1999) who find that students who scored in the bottom quartile (and thus find themselves in the $12th$ percentile, on average) in tests regarding humor, grammar, and logic skills, believe themselves to be in the $63rd$ percentile of the distribution, and Ben-David et al. (2013) who show that only 36.3% of the time the S&P500 falls into the 80% confidence interval provided by CFOs of mid-size and large U.S. corporations. Further examples include Weinstein (1980) and Slovic (2000). The literature mainly uses three definitions of overconfidence: (1) the overestimation of the probability of positive events; (2) the overestimation of one's performance compared to others; and (3) the overestimation of the precision of one's information. The model and identification approach of this paper correspond to the first definition.

[7] An exception is the work by Attanasio et al. (2017), who estimate Euler equations for consumption using subjective expectation data.

The majority of these studies use questions only allowing for yes-no answers to elicit expectations. Manski (1990) shows that even in the absence of aggregate shocks, binary expectations questions are ill-equipped for investigating the hypothesis of rational expectations.[8] In addition, nearly all these questions mix pure expectations of exogenous events with preferences that prevent a clear distinction between these two factors. In contrast, the model and identification strategy presented in this paper do not rely on questions to elicit expectations and, therefore, does not suffer from these problems. It also allows clearly differentiating between biased expectations of exogenous future events and changes in preferences.

A stronger focus on biased beliefs of future employment prospects represents the work of Spinnewijn (2015). He examines the optimal unemployment insurance design when job seekers overestimate their chances of finding employment. In addition to a theoretical analysis of how to adjust the Baily formula for optimal unemployment insurance (Baily, 1978; Chetty, 2006) in the presence of overconfidence, Spinnewijn (2015) calibrates a job search model with various degrees of biased expectations. He finds that overconfident agents are less responsive to future incentives and shows that it can be optimal for unemployment benefits to increase over time. Complementing this work, this paper concentrates on the individual career costs of mothers in a life-cycle framework. Since the majority of female career breaks are family-related, adjusting maternity-leave policies might be more effective for women than adjusting unemployment insurance.

Another empirical investigation of overconfidence and its consequences in labor supply contexts is Hoffman & Burks (2017). They investigate the overconfidence in productivity by truck drivers, finding it contributes to fewer employees quitting. Overall, this causes welfare to increase, since the companies face large initial training costs when hiring new drivers. I extend this research by discussing the effect of overconfidence on the career development of mothers. In contrast to Hoffman & Burks (2017), my results indicate that there can be substantial costs when individuals are too optimistic about their future employment possibilities.

This paper also contributes to the literature focusing on employment and maternal welfare in a life-cycle context. Adda et al. (2017), using a life-cycle model of occupational choice, find that

[8] A short example should illustrate this statement. Assume a single event A occurs with the probability of 51%. If the event is realized, a subject will work the next period, otherwise she will spend time in home production. If asked if they will expect to be working next period, all subjects will answer with "yes," since "no" is more unlikely. On average, this results in a discrepancy between the stated expectations and realizations of 49 percentage points. For a more general discussion of the importance of expectations in economics and their measurement see Manski (2004).

family-oriented women already choose occupations that are family friendly but not necessarily well paid. They estimate the cost of having children to be about 35% of lifetime income. Some of these costs also stem from lost earnings and depreciation of human capital during career breaks. Blundell et al. (2016) estimate a model of human capital accumulation and depreciation that points to very low human capital accumulation in part-time employment and, therefore, stagnating careers for mothers who tend to work part-time. Employing a similar model of life-cycle labor supply, I extend their findings by dividing career costs into expected and unexpected ones. While anticipated career costs do not necessarily justify policy interventions when markets are close to perfect, biased expectations can be regarded as market imperfections and, thus, make a stronger case for additional regulations. An example of a more harmless intervention is the direct provision of information, for example, in the form of letters. These seem to work well in some fields of public economics (see for example Bhargava & Manoli (2015), and Duflo & Saez (2003), and in the German context Dolls et al. (2016)).

Finally, this paper adds to the growing literature of behavioral public economics. Because optimal policy design depends on the behavior of individuals, ignoring behavioral insights may lead to ineffective policy recommendations. Some behavioral insights can also lead to more efficient policies, such as providing additional information or commitment devices, which might otherwise have been ignored. An example in the context of labor supply is DellaVigna et al. (2017), who exploit a reform of the unemployment benefit system in Hungary, showing that job seekers have reference-dependent preferences. They argue that in this case a multi-step unemployment insurance is optimal. Other examples are DellaVigna & Paserman (2005) and Chan (2017), who investigate time-inconsistent preferences in the form of hyperbolic-discounting. The former find that measurements of the impatience of job seekers and their respective unemployment lengths are in line with the hyperbolic-discounting model. Chan (2017) identifies discounting parameters using data from a field experiment. He finds evidence for a welfare-trap: individuals, who are not currently employed, postpone their decision to start working due to time-inconsistent behavior. I extend this literature by determining how expectations might contribute to the length of non-employment durations.

The paper proceeds as follows. Section 2 discusses the institutional framework. Section 3 describes the data. Section 4 presents some descriptive characteristic of the data and provides suggestive evidence for the biased expectations of future employment prospects. Section 5 develops the structural life-cycle model. Section 6 discusses the identification and estimation of the model parameters, in particular the identification of beliefs. Section 7 presents the results and discusses their implications. Section 8 concludes.

2 Maternity Leave Policy in Germany

German maternity leave regulations provide an ideal setting for the identification of expectations of future employment possibilities. Several policy reforms extended the period granting mothers the right to return to their previous work positions, which provides exogenous variation for identification. In total, I exploit multiple major expansions of maternity leave coverage between 1986 and 1993, which I summarize as three major policy regimes.[9] The objective of these reforms was twofold. First, they intended to encourage mothers to spend more time with their children during their early development. Second, they aimed to increase maternal labor market attachment, since longer employment protection was viewed as an instrument to ease returning to the labor market. Since the identification approach relies on the exogenous variation created by these reforms, a more detailed discussion of the maternity leave system and its changes is discussed in the following.

Starting in the late-1960s and through 1986, mothers were entitled to 14 weeks of paid leave around childbirth, during which women were generally not allowed to work. While on leave, employers could not dismiss mothers and had to provide a comparable job to the previously held position for mothers returning within leave. During the 14 weeks, women received their average income of the three months before entering maternity leave, resulting in an income replacement rate of 100%. The core of this law is still effective in 2017,[10] with later reforms mainly changing the regulations after 14 weeks. In the late-1970s, the first major reform extended maternity leave coverage, lengthening the employment protection period to six months after childbirth and introducing a new maternity leave payment for the time between the end of the 14$^{\text{th}}$ week and the end of the 6$^{\text{th}}$ month. In this period, women, who were employed before having a child, received DM 750[11] per month.
Reforms used for identification started in 1986, with table 1 providing an overview. The first reform expanded the employment protection and maternity benefit period from six to ten months at the

[9] The summary of the parental leave reforms through 1985 are mainly based on Zmarzlik et al. (1999). For later reforms, see Bundeserziehungsgeldgesetz [BErzGG] [Federal Child-Raising Benefit Act], Dec. 6, 1985, BGBl.I at 2154 (F.R.G.) and its changes through its abolition in 2007. This paper concentrates on West Germany.

[10] Minor reforms specified more precisely the conditions under which mothers are allowed to work during this period.

[11] This is equivalent to $ 758 in 2017.

beginning of 1986, then extending it to 12 months in January 1988.[12] Maternity payments from week six to week eight remained at an income replacement of 100% or DM 600[13] if the mother was previously unemployed. Between three and six months, maternity benefits declined from DM 750[11] to DM 600[13] per month. From the seventh month to the 10$^{\text{th}}$ month (and later 12$^{\text{th}}$ month), the amount of maternity benefits was means tested and depended on the family income during the two years prior to childbirth. Around 84% of individuals were eligible for the full benefits (Schoenberg & Ludsteck, 2014). I summarize these conditions as forming policy regime I, which provides one full year of employment protection and maternity benefits.

Table 1: Parental leave reforms from 1986 until 2006

	Month, Year (1)	Emp. Prot. (2)	Maternity Benefits (3)
Regime I	January, 1986	10 months	3-6 months DM 600,[13] 7-10 months means tested
	January, 1988	12 months	up to 12 months
Regime II	July, 1989	15 months	up to 15 months
	July, 1990	18 months	up to 18 months
Regime III	January, 1992	36 months	up to 18 months
	January, 1993		up to 24 months
	January, 2007	maternal benefits are related to previous earnings	

A further expansion of employment protection and the maternity benefit period from 12 months to 15 months took effect in July 1989, followed by another extension to 18 months in July 1990. These reforms are summarized in regime II, providing 1.5 years of employment protection and maternity benefits. In January 1992, the employment protection period was further extended to a total of three years. In contrast, the maximum maternity payment period initially remained constant at 18 months, before being extended to two years a year later. Minor changes in family

[12] Additionally, paternity leave was introduced. However, between 1987 and 1994 only about 1% of fathers took parental leave (Vaskovics & Rost, 1999).

[13] This is equivalent to $ 606 in 2017.

policy were introduced in 2001, but the core regime of 1993 still continued.[14] This forms regime III, which provides 3 years of employment protection and 2 years of maternity benefits. These policies did not noticeably change, before a major reform of maternity benefits in 2007, which changed benefits from a lump-sum payment to an income replacement based on pre-birth earnings.

3 Data

For estimations, I use longitudinal data from the German Socio-Economic Panel Study (SOEP) covering 1986 through 2006 (see Wagner et al., 2007, for a description of the SOEP).[15] Starting in 1984, the SOEP interviews private households and persons in Germany. Each year, all household members older than 16 are interviewed annually, conditioned on their voluntary collaboration. New additions to a household, including partners and children, also remain in the sample, even after leaving the household. The original SOEP sample was expanded with several booster samples over the years and questionnaires cover a wide range of topics, including details on demographics, education, labor market dynamics, earnings, and other income, among others.

While the SOEP interviews individuals on a yearly basis, it asks participants to fill out a monthly calendar for the previous year. In particular, individuals are asked about last year's employment history, which allows me to construct a semi-annual data set by combining the current year's questionnaire with information from the questionnaire of the following year. A particularly useful feature is that the SOEP has a possible option "maternity leave" for the monthly employment state. That previously employed mothers consistently use this option emphasizes that the employment protection is a well-known and well-understood policy to mothers. The SOEP also asks newly surveyed individuals, older than 16, to fill out a specific questionnaire collecting information about their life before they were included in the sample. This enables me to collect information on each individual's age when finishing their education and starting their work life.

I restrict the sample to women and, when applicable, their partners between the age of 18 and 50,[16] having no university degree after finishing education. The education limitation is mainly due

[14]There was a minor change in the maternity benefits in 1994. For the first six months, benefits were also means tested. For married couples, the threshold was DM $100,000$, for singles DM $75,000$ for receiving the full benefits during the first six months.

[15]I also use the wave corresponding to 2007, since it includes responses addressing 2006.

[16]Some exogenous processes estimations, include women until the age of 70 in order to have more robust estimates for the later years.

to the small number of highly educated women in the sample having children around the time of the maternity leave reforms used for identification. Because some reforms took place before the reunification of Germany, I exclude individuals living in East Germany. Additionally, I exclude self-employed women and women working in the public sector. Self-employed individuals are not impacted by the reforms, because they do not have an employer who has to guarantee them their job for a given period. In contrast, individuals working in the public sector might enjoy more generous maternity leave conditions, especially longer periods of employment protection.[17] For similar reasons, women who are subject to least one of the following criteria are also not included in the sample: living outside Germany, being severely disabled, or having at least one multiple birth. Missing information on the age of their children, their labor market entry age, or their labor market experience also leads to exclusion.

Some further data cleaning and labeling is worth highlighting. The labor market experience for a given year is constructed by combining the answers of a working history questionnaire and the recorded employment state of follow up interviews. Part-time and full-time experiences are separately measured. Wages are defined as gross monthly earnings divided by actual working hours during the same period. Since the model does not include any macro economic processes, all money values are deflated using the Consumer Price Index and the base year 2000 (Federal Reserve Bank of St. Louis, 2016). To reduce the importance of measurement error in wages, the wage distribution is trimmed at the 4^{th} and 98^{th} percentiles, from below and above, respectively.[18]

The resulting data set is an unbalanced panel in which individuals enter and leave the panel at various ages. In total there are $3,944$ women. Approximately 55% of these are observed for more than 5 years, about 20% for more than 10 years. Additionally, I observe $1,510$ births and a total of $2,934$ children aged 18 or younger. In total, the sample has $49,924$ semi-annual observations. Table 2 shows the distribution of family types for three age groups. Women tend to get married before turning 30. This seems to be an important factor for having children, since the number of single mothers are rather low, with a peak of 8% at age 35. At this age, 83% of women have had a child, potentially interrupting their working career. Ninety percent of the mothers are married and, thus, might be able to rely on income of their husband.

[17]Note that to qualify for the more generous maternity leave conditions in the public sectors, individuals must to have been employed regularly in this sector directly before having their child. Therefore, mothers are not able to receive these more generous conditions when switching sectors *after* having had their child.

[18]After trimming, the lowest hourly wage is €4.21 and the highest wage is €25.72.

Table 2: Distribution of family types at different ages

	Mothers		Non Mothers		Number of
	Singles	Marriage	Singles	Marriage	Observations
	(1)	(2)	(3)	(4)	(5)
women aged 25	0.04	0.38	0.24	0.33	937
women aged 30	0.06	0.65	0.09	0.20	925
women aged 35	0.08	0.75	0.06	0.11	923

Finally, since the identification exploits the different maternity leave regimes, an overview of the number of observations in the three regimes is helpful. It is provided in table 3. Although the SOEP is a survey and regime II was only in place for 2.5 years, I observe 110 distinct women with a child under the age of 3 for the second regime. For these women, 999 labor supply decisions are recorded. For regimes I and III, I observe 359 and 854 women with young children, respectively. These make $1,395$ and 6607 labor supply choices.

Table 3: Observations per regime

	women	decisions
	(1)	(2)
Regime I	359	1395
Regime II	110	999
Regime III	854	6607

Notes: Column 1 represents the number of women observed in the respective regime who have a child under the age of 3. Column 2 represents the number of decisions observed for these women.

4 Suggestive evidence for overconfidence

The SOEP questionnaire allows for a first test of biased beliefs of future employment opportunities. Since 1999, all non-employed subjects who indicated that they might seek employment in the future are requested to state probabilities of future life events, including the likelihood of being employed within the next two years.[19] Knowing the month of the interview, it is possible to track individuals who stated a probability to enter employment and determine if they found employment within two years. Comparing the average stated probability and the average of individuals who have found employment provides a first estimate of a bias in expectations. As table 4 shows, individuals systematically overestimate their future attachment to the labor market.[20] It is important to note that due to the limited number of mothers answering these questions, the table includes also women without children.

The first column shows the average stated likelihood and the average actual realization of the whole sample. Columns two to four list the values for women who stated a likelihood of being in employment within the next two years of at least 30%, 50%, and 80%, respectively. To list values of these groups seems important, since Kassenboehmer & Schatz (2017) show that low stated probabilities are primarily driven by individuals who are long-term unemployed and lost faith in their ability finding employment.

In table 4, the average stated probability is provided in row 1, the actual realization of the same individuals in row 2. Row 3 states by how much individuals overestimate their chances to find employment within two years. Row 4 provides the p-value of testing the hypothesis that the difference between row 1 and 2 is zero, while the last row reports the number of observations for each respective group. The critique of Manski (1990) that standard expectation data is not well equipped to investigate the question of rational expectations does not apply in this case, since individuals are explicitly asked to state the probability using a Likert-type scale with a range of 11 values. Manski's critique is mostly applicable to questions with binary answers, since these are more challenging for estimating underlying probabilities.[21]

In addition, the data is collected between 1999 and 2006, a period during which Germany's unemployment rate did not fluctuate much, staying between 9% and 12% percent. Two years in which the question about future employment expectations was asked were followed by a decline in the unemployment rate, while the other two years were followed by a recession.[22] Thus, it is

[19]For the exact questions, see appendix A.1.

[20]See also Kassenboehmer & Schatz (2017) who find similar results in a sample including men.

[21]The Likert-type scale has the range 0% to 100% in 10 percentage points steps. In the worst case, under rational expectations, individuals would estimate their likelihood only slightly above the median between two points on the scale and, thus, always choose the higher value. Hence, a deviation below 5% does not necessarily provide enough evidence to reject the null hypothesis of rational expectations. Even when subtracting 5% off all stated likelihoods, the differences stated in table 4 are still significant at the 1% level.

[22]For a detailed overview of the unemployment rate during the time of the interviews see appendix A.2

Table 4: Employment expectations and realizations

	Average	Stated >= 30	Stated >= 50	Stated >= 80
	(1)	(2)	(3)	(4)
Stated	45.44 %	66.55 %	72.89 %	92.75 %
Actual	35.95 %	45.93 %	48.75 %	58.78 %
Overestimation	26.40 %	44.89 %	49.52 %	57.79 %
p-value	0.0000	0.0000	0.0000	0.0000
Observations	1079	690	569	279

Notes: Row 1 represents the stated probability to be in employment within the next two years, row 2 states the real percentage of individuals having found employment within two years, row 3 shows by how much individuals overestimated the probability on average, row 4 denotes the p-value of the hypothesis that there is no difference between row 1 and row 2 and row 5 shows the number of observations. The original question reads as follows: "How likely is it that you will start paid work within the next two years?" Only subjects who stated that they might want to work in the future are asked this question. The answers are recorded on an 11-point Likert-type scale from 0 to 100 percent. Individuals for whom I can neither observe the length of their unemployment spell nor that they were unemployed for more than two years are excluded. Survey weights are used.

plausible to assume that the differences in expectations and realizations are not driven by aggregate shocks.

The table unveils two important aspects. First, it shows that only a minority (35.95%) of interviewed women enter employment within two years and second, individuals overestimate the likelihood of finding employment by 26%, on average. This gap between the stated likelihood and the observed outcomes widens monotonically with higher stated probabilities. For individuals who stated a probability over 50%, the average prediction is about 73%, but in reality only 49% find employment within two years, causing an overestimation of about 48%. A second aspect is that expectations are not random, as there is a positive correlation between stated preferences and realizations. Individuals who state a higher likelihood to find employment, also have a higher probability to enter employment within two years. It seems that individuals can, to a certain degree, predict their likelihood in relation to others, but systematically overestimate the likelihood of finding employment on average.

5 Model

Although, there is some suggestive evidence showing that women indeed substantially overestimate their future employment possibilities, it is unclear how accurate stated probabilities are, since even college students often have trouble grasping the concept of probability (Garfield & Ahlgren, 1988). Another strategy to test the hypothesis of rational expectations is to rely on a revealed preferences approach, identifying expectations from actual employment choices. To do so, a standard dynamic discrete choice model of labor supply is developed, enhanced by explicitly modelling future employment expectations. The presented model relates strongly to the model in Blundell et al. (2016), who also investigate female labor supply over the life-cycle.[23] In addition to permit identification of expectations from choice data, the model can be used to analyze the costs of biased beliefs; an exercise, challenging to perform based on the descriptive analysis presented in the previous section.

5.1 Outline of the model

Figure 1 provides an overview of the model's general life-cycle process. The main focus lies on the working life of females between the ages of 18 and 50. Since the age at which individuals leave education is considerably heterogeneous in Germany, the model begins after the respective individual has finished education and enters the working life.

As figure 1 shows, all women enter the model's life-cycle element after finishing their education. Each decision period lasts for six months and begins with the determination of the state space of the current period. This includes the forming and termination of marriages, the birth of children, and the realization of wage and taste shocks. Similar to Blundell et al. (2016), partnerships and children are not modeled as explicit choices, but as exogenous stochastic processes. These processes depend on the characteristics of the women, including age and current family characteristics. When individuals maximize their expected lifetime utility, they account for the possibilities of partnerships and children.[24]

[23] Besides explicitly modelling expectations, the major difference to the following discussed model is that labor market frictions in the form of job offers and layoffs are not explicitly modeled in Blundell et al. (2016). In contrast, the authors integrate a savings decision in their model. Besides Blundell et al. (2016), there is a long history for dynamic life-cycle models of labor supply, see for example Heckman & Macurdy (1980), Eckstein & Wolpin (1989), Van der Klaauw (1996), Attanasio et al. (2008), and Adda et al. (2017).

[24] Not modeling marriages and the arrival of children as choices is mainly due to the additional computational burden when doing so. An important limitation resulting from this modelling choice is that counterfactual simulations are assumed to not impact these processes. There is also a long history of modeling partners and children in this way, see, among others, Van der Klaauw (1996), Sheran (2007) and Blundell et al. (2016).

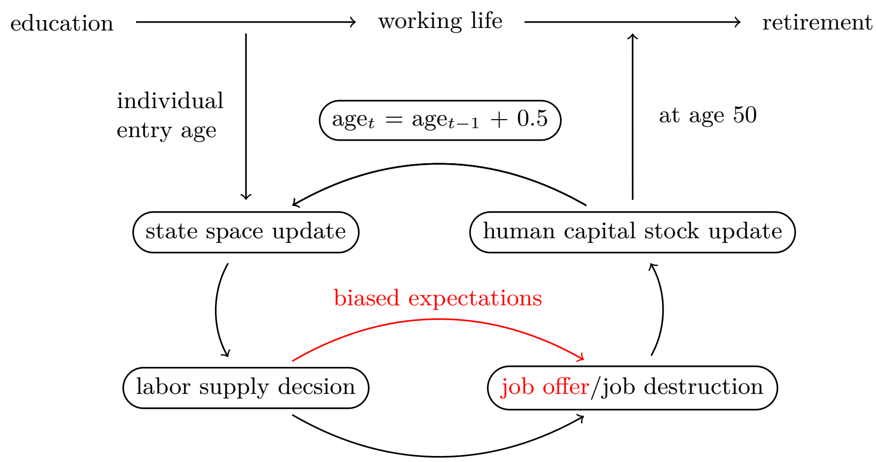

Figure 1: Outline of the model

After the state space for the current period is set, each woman chooses the number of hours she wants to supply for the current period. The possible supplied hours are discretized into three categories, non-employment (0 hours per week), part-time employment (20 hours per week), and full-time employment (40 hours per week). The realization of the hours choice depends on the labor market and the woman's previous employment state. If a woman was not employed in the previous period, she needs to receive a job offer if she desires to work in the current period. If a woman was employed in the previous period, she might lose her employment involuntary due to plant closure or other external factors. In this case, she can work neither full-time nor part-time in the current period and must await the next period for possible reemployment. An important feature of the model is that women have expectations of the probability of receiving a job offer in a future period. Although these expectations can align with the true job offer probability, individuals are allowed to systematically under- or over-estimate these probabilities. In contrast, all other probabilities, for instances about partnerships and children, are restricted to align with the true probabilities.

At the end of each period, the on-the-job human capital stock is updated. It is a crucial factor for determining the wage that an individual woman can realize on the labor market. All individuals start with an on-the-job human capital stock of zero, since it is only gained through employment. Similar to Blundell et al. (2016), human capital might grow with different rates for full-time and part-time employment. Furthermore, human capital depreciates with every period spend outside of employment. Each periods ends with the update of the on-the-job human capital. The outlined process repeats until the age of 50, which is the last period of the model.

5.2 The structural model

This subsection provides further details about the functional form assumptions. Individuals must make a labor supply decision (l_t) each period, depending on their characteristics after entering the working life phase of the model. The characteristics include the age (t), on the job human capital (e_t), the employment state of the last period (l_{t-1}), the presence of a partner (p_t), the presence of children (cd_t), age of the youngest child (ac_t), current policy regime (r_t), and the employment protection state (jp_t). In principle, they can choose between non-employment ($l_t = NE$), part-time employment ($l_t = PT$), and full-time employment ($l_t = FT$).[25]

Flow utility. The instantaneous utility of a choice depends on its consumption opportunities and its leisure time. Consumption and the utility from leisure are allowed to vary with the presence of a partner, the presence of children, and the age of the youngest child. I assume the instantaneous utility is non-separable between consumption and leisure, but utility is separable over time. The functional form is given by

$$u_{i,t} = \frac{(c_{i,t}/\bar{c}_{eq} - 1)^{(1-\gamma_c)} - 1}{1-\gamma_C} \times \exp\Big(U^L\left(l_{i,t}, p_{i,t}, cd_{i,t}, ac_{i,t}\right)\Big) + \varepsilon_{i,t}, \tag{1}$$

where $c_{i,t}$ denotes the consumption and $\bar{c}$ an equivalence scale[26] that controls for the members of the household. The CRRA-parameter γ_c represents the risk aversion. $U^L(\cdot)$ represents the utility from leisure varying with education and family demographics. It is normalized to 1 if a woman does not work. Finally, the choice specific shock $\varepsilon_{i,t}$ is independently and identically distributed over time and choices with a type-1 extreme value distribution with zero-mean. The utility of leisure is normalized to 1 if the individual is not working in the current period. The leisure preferences vary with working hours and education, additionally depending on the presence of a partner and

[25] I assume 260 paid working days in a given year, which equals 130 working days in a half-year. Part-time employment is standardized to be 20 working hours in a week (520 hours a half-year), full-time employment to be 40 working hours in a week (1040 working hours a half-year). Both hour values are the median hours worked in the sample, when subjects stated that they are working part-time or full-time, respectively.

[26] I assume that $\bar{c} = 1$ for single women without children, $\bar{c} = 1.4$ for single mothers, $\bar{c} = 1.6$ for couples without children, and $\bar{c} = 2$ for couples with children.

children, as shown in equation (2).

$$
\begin{aligned}
U^L\left(l_{i,t}, p_{i,t}, cd_{i,t}, ac_{i,t}\right) = &\sum_{l' \in \{PT, FT\}} \left(\gamma_{l'} \mathbb{1}_{[l_{i,t}=l', cd_{i,t}=0]} + \gamma_{l',p} \mathbb{1}_{[l_{i,t}=l', p_{i,t}=1]}\right) \\
&+ \sum_{l' \in \{PT, FT\}} \mathbb{1}_{[l_{i,t}=l', cd_{i,t}=1]} \left(\gamma_{l',ac_0} + \gamma_{l',ac_1} ac_{i,t} + \gamma_{l',ac_2} ac_{i,t}^2\right) \\
&+ \sum_{l' \in \{PT, FT\}} \sum_{ac' \in \{0, 0.5, \ldots, 3.5\}} \mathbb{1}_{[l_{i,t}=l', ac_{i,t}=ac', cd_{i,t}=1]} \gamma_{l',ac'} \\
&+ \gamma_{PT,state} \mathbb{1}_{[l_{i,t}=l_{i,t-1}=PT]} + \gamma_{FT,state} \mathbb{1}_{[l_{i,t}=l_{i,t-1}=FT]}
\end{aligned}
\tag{2}
$$

The leisure preferences, depending on the age of the youngest child, are modeled in a quadratic manner, but allowed to deviate for very young ages. This allows capturing irregularities in preferences that cause mothers to return during their employment protection and are important to control for, since the identification strategy relies on the excess mass of mothers returning due to their low expectations of future employment opportunities, when controlling for preferences. The last two terms capture a possible state dependency since, in the data, women rarely switch directly from part-time to full-time employment or vice versa.

Wages and Human Capital. The decision to work and the resulting hours choice depend on the consumption opportunities of the respective choice. Income from employment is driven by the following wage process:

$$
\ln(w_{i,t}) = \ln(\gamma_{w,\,const.}) + \gamma_{w,e} \ln(e_{i,t} + 1) + \xi_{i,t} \tag{3}
$$

The hourly wage rate depends on a constant and accumulated on-the-job human capital. $\xi_{i,t}$ is to be assumed a measurement error that follows a normal distribution with standard deviation σ_ξ. Wage differences over time are driven by on-the-job human capital. It evolves in the following manner:

$$
e_{i,t} = \begin{cases} e_{i,t-1}(1-\eta) & \text{if } l_{i,t-1} = NE \\ e_{i,t-1}(1-\eta) + \lambda & \text{if } l_{i,t-1} = PT \\ e_{i,t-1}(1-\eta) + 0.5 & \text{if } l_{i,t-1} = FT \end{cases} \tag{4}
$$

Human capital at the end of each period depends on the previous period's human capital and the employment state of the current period. In each period, the on-the-job human capital depreciates with the rate $(1-\eta)$.[27] Accumulation depends on the working hours with potentially different gains for part-time and full-time employment. Being in a model with a semi-annual decision period, the gain of full-time employment is normalized to be 0.5. The gain from part-time employment is estimated in order to not restrict the model to a specific ratio in wage growth between the two employment states. All parameters of the human capital process are education specific.

Budget Constraint. Given the labor supply decision and the wage process, consumption is determined by:

$$\begin{aligned} c_{i,t}(l_{i,t}, p_{i,t}, ac_{i,t}) = 520 \times \Big(w_{i,t} - \mathbb{1}_{\{cd_{i,t}\}} cc(ac_{i,t})\Big) \\ \times \left(2 \times \mathbb{1}_{\{l_{i,t}=FT\}} + \mathbb{1}_{\{l_{i,t}=PT\}}\right) \\ + \mathbb{1}_{\{p_{i,t}\}} earn^p_{i,t} - TT(earn^w_{i,t}, earn^p_{i,t}, cd_{i,t}, ac_{i,t}) \end{aligned} \tag{5}$$

where $earn^w_{i,t}$ and $earn^p_{i,t}$ stand for the gross labor earnings of the woman and her partner, respectively. The function $cc(\cdot)$ stands for the childcare costs, which are taken from the data and depend on the age of the youngest child.[28] For each hour that the woman works, she needs childcare for children under the age of 6. $TT(\cdot)$ represents the German tax and transfer system. I model all key features of the German tax and transfer system. In particular, joint taxation, unemployment benefits, social assistance, and childcare benefits are modeled carefully, since they might strongly affect the financial incentives to work.

Job offers and expectations. Women face labor market frictions when seeking employment. To switch from non-employment to part-time or full-time work, women must receive a job offer for the respective number of hours. An offer is also necessary to switch from part-time to full-time employment or vice versa.[29] The arrival probability of these offers depend on the lifetime under- or non-employment periods ($\rho_{i,t}$), which are given by the age minus the labor force entry age and the current human capital stock:

$$\rho_{i,t} = t - t_{\text{labor force entry}} - e_t. \tag{6}$$

[27] At the start of the working life, every individual is assumed to have zero on-the-job human capital.

[28] I follow the approach of Wrohlich (2011) by including individuals without positive childcare costs when computing the average expected childcare costs. One hour of care costs €1.82 for children under the age of 3 and €1.15 for children between the age of 3 and 6.

[29] No offer is needed to keep working part-time or full-time.

Given $\rho_{i,t}$, the job offer probabilities are given by

$$\pi^{PT}(l_{t-1},\rho_t)=\begin{cases}\dfrac{\exp\left(\gamma_{PT}+\gamma_{PT,\rho,1}\rho_{i,t}+\gamma_{PT,\rho,2}\rho_{i,t}^2\right)}{1+\exp\left(\gamma_{PT}+\gamma_{PT,\rho,1}\rho_{i,t}+\gamma_{PT,\rho,2}\rho_{i,t}^2\right)} & \text{if } l_{t-1}\neq PT \text{ and } jp_{i,t}=0\\ 1 & \text{if } l_{t-1}=PT \text{ or } jp_{i,t}=1\end{cases}\tag{7}$$

$$\pi^{FT}(l_{t-1},\rho_t)=\begin{cases}\dfrac{\exp\left(\gamma_{FT}+\gamma_{FT,\rho,1}\rho_{i,t}+\gamma_{FT,\rho,2}\rho_{i,t}^2\right)}{1+\exp\left(\gamma_{FT}+\gamma_{FT,\rho,1}\rho_{i,t}+\gamma_{FT,\rho,2}\rho_{i,t}^2\right)} & \text{if } l_{t-1}\neq FT \text{ and } jp_{i,t}=0\\ 1 & \text{if } l_{t-1}=FT \text{ or } jp_{i,t}=1\end{cases}\tag{8}$$

Equations (7) and (8) highlight that no job offer is required to continue working part-time or full-time. In addition, the maternity leave entitle women with the right to return to employment, independent of the hours. It is possible that women systematically over- or underestimate these offer probabilities and it is assumed that individuals do not update their expected job offer rate over time.[30] With $\widetilde{\pi}^{\cdot}(l_{t-1,e_{t-1},t-1})$ standing for the expected job offer rate, the following relation between the expected and the true job offer rate is given:

$$\begin{aligned}\widetilde{\pi}^{PT}(l_{t-1},\rho_t)&=\begin{cases}\alpha^{PT}\,\pi^{PT}(l_{t-1},\rho_t) & \text{if } \pi^{PT}(l_{t-1},\rho_t)<1\\ 1 & \text{if } \pi^{PT}=1\end{cases}\\ \widetilde{\pi}^{FT}(l_{t-1},\rho_t)&=\begin{cases}\alpha^{FT}\,\pi^{FT}(l_{t-1},\rho_t) & \text{if } \pi^{FT}(l_{t-1},\rho_t)<1\\ 1 & \text{if } \pi^{FT}=1\end{cases}\end{aligned}\tag{9}$$

$$\text{where}\qquad \alpha^{PT}\in\left[0,\frac{1}{\pi^{PT}(l_{t-1},\rho_t)}\right],\ \alpha^{FT}\in\left[0,\frac{1}{\pi^{FT}(l_{t-1},\rho_t)}\right].$$

The parameters α^{PT} and α^{FT} determine the degree of deviation from the true job offer rate. They can never fall below zero, since this would result in a negative expected job offer rate. Similarly, both parameters must not be larger than the inverse of the true job offer arrival rates since the

[30]Due to the rare event of being non-employed and then re-entering employment, it seems plausible that individuals do not have many opportunities to learn about the real job offer rate over the life-cycle.

highest expected percentage is not larger than 100 percent. Individuals do understand the concept of maternity leave and know that they have the right to return to their previous position. Depending on the size of α^{PT} and α^{FT}, the individuals might have rational expectations, underestimate, or overestimate the true job offer rate:

$$\begin{aligned} \text{rational expectations if } \alpha^{\{PT,FT\}} &= 1 \\ \text{underestimation if } \alpha^{\{PT,FT\}} &< 1 \\ \text{overestimation if } \alpha^{\{PT,FT\}} &> 1 \end{aligned} \tag{10}$$

The nesting of rational expectations in the model allows for straightforward testing of the hypothesis of non-biased expectations, by testing the hypothesis of $\alpha^{\{PT,FT\}} = 1$.

Job loss. When employed in the previous period, a woman can also involuntary lose her employment. Provided the woman worked in the previous period, there is an exogenous probability that the plant closes, denoted in the model as $\delta(\omega_t)$. In this case, she is not able to choose employment in the current period and must await a job offer in the next period if she wants to re-enter employment. This probability is estimated outside the model using information provided in the SOEP sample. The questionnaire asks participants for the particular reason when a transition from employment to non-employment occurs. Among the answer options, only involuntary reasons like layoffs and plant closures are used to estimate the job loss probability.

Family dynamics. The birth of children, along with the formation and termination of partnerships, are modeled as exogenous stochastic processes depending on the woman's age and current family demographics. The probability of having a first child differs from the probability of having additional children.[31] In the model, and in line with Blundell et al. (2016), only the age of the youngest child is important, thus whenever a new child is born, the age of the youngest child is reset to zero. Children live in the household until they turn 18. Beginning a new partnership depends only on age, while separations also depend on the presence of a child and their age. Partners contribute to the household consumption and affect the women's leisure preferences. To keep the computational burden manageable, the partners' earnings are modeled to depend on the characteristics of the woman, including her age and family characteristics.[32] Agents in the model know and

[31] Since the model's decision period is a half-year, women are not able to have an additional child if the youngest child has not reached the age of one.

[32] This approach is similar to Van der Klaauw (1996), Sheran (2007) and Adda et al. (2017).

accommodate for these probabilities when forming expectations of future periods.

5.3 Maximizing expected lifetime utility

Given the preferences, the labor market frictions, and the external processes, women maximize their expected lifetime utility each period. In a given period t, this utility is formally given by

$$\max_{\{l_t, l_{t+1}, \ldots, l_T\}} V_t(l_t, l_{t+1}, \ldots, l_T, \omega_t) = u(l_t, \omega_t) + \mathbb{E}\left[\sum_{\tau=t+1}^{T} \beta^{\tau-t} u(l_\tau, \omega_\tau) \middle| \omega_t\right] \tag{11}$$

where the index of i is dropped for the ease of notation. The parameter β represents the discount factor, $\mathbb{E}[\cdot]$ the expectation operator, and ω_t a realization of the state space Ω_t in period t. The state space is defined as

$$\Omega_t = \{e_t, l_{t-1}, cd_t, ac_t, p_t, jp_{t-1}, r_t\}$$

Having specified the lifetime utility, and assuming the separability between the choice-specific error term and the rest of the utility function, the model can be represented in a two period decision process characterized by the Bellman (1957) equations (12) and (13).

$$
\begin{aligned}
&\text{for } t<T: \\
&v_t(l_t,\omega_t) \\
&\quad = u^*(l_t,\omega_t) + \varepsilon_{l_t,t} \\
&\qquad + \beta \sum_{\substack{\omega_{t+1} \\ \in \Omega_{t+1}}} \Bigg\{ \delta(\omega_t)\mathbb{E}\left[v^*_{t+1}(NE,\omega_{t+1}) + \varepsilon_{NE,t+1}\right] + (1-\pi^L(\omega_t)) \Bigg(\\
&\qquad\qquad + \tilde{\pi}^{PT}(\omega_t)(1-\tilde{\pi}^{FT}(\omega_t))\mathbb{E}\left[\max_{\substack{j\in \\ \{NE,PT\}}} \left\{v^*_{t+1}(j,\omega_{t+1}) + \varepsilon_{j,t+1}\right\}\right] \\
&\qquad\qquad + \tilde{\pi}^{FT}(\omega_t)(1-\tilde{\pi}^{PT}(\omega_t))\mathbb{E}\left[\max_{\substack{j\in \\ \{NE,FT\}}} \left\{v^*_{t+1}(j,\omega_{t+1}) + \varepsilon_{j,t+1}\right\}\right] \\
&\qquad\qquad + \tilde{\pi}^{PT}(\omega_t)\tilde{\pi}^{FT}(\omega_t)\mathbb{E}\left[\max_{\substack{j\in\{NE, \\ PT,FT\}}} \left\{v^*_{t+1}(j,\omega_{t+1}) + \varepsilon_{j,t+1}\right\}\right] \\
&\qquad\qquad + (1-\tilde{\pi}^{PT}(\omega_t))(1-\tilde{\pi}^{FT}(\omega_t))\mathbb{E}\left[v^*_{t+1}(NE,\omega_{t+1}) + \varepsilon_{NE,t+1}\right] \Bigg) \\
&\qquad\qquad \Bigg\} q(\omega_{t+1}|l_t,\omega_t)
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
&\text{for } t=T: \\
&\quad v_t(l_T,\omega_T) = u^*(l_T,\omega_T) + \varepsilon_{l_T,T}
\end{aligned} \tag{13}
$$

where $q(\omega_{t+1}|l_t,\omega_t)$ denotes the probability of arriving at state space ω_{t+1} given choice l_t and state space ω_t, and $u^*_{i,t}$ the utility function without the choice specific error term, i.e. $u^*_{i,t} \equiv u_{i,t} - \varepsilon_{i,t}$. Similarly, $v(\cdot)^* \equiv v(l_t,\omega_t) - \varepsilon_{l_t,t}$ denotes the value function without the choice specific error term. Furthermore, if the current choice is non-employment ($l_t = NE$), the job loss probability is zero ($\delta(\omega_t) = 0$), since women can only lose their job if they are employed. The biased expectations of the future job arrival rates enter only the value function, since they represent what the individual beliefs about future possibilities.

Two assumptions help with the formulation of the stated value functions. First, I assume that individuals do not know that their expected job offer probability might differ from the actual offer rate and, second, they do not update their expected job offer probability over the life-cycle. This causes individuals to treat the expected employment probability as given when maximizing their expected lifetime utility. As a result, there is no correlation between the expected job offer rate and the expected choice specific error component. Additionally, I assume that mothers fully understand the institutional settings and know that they receive employment protection when they qualify for it. Thus, mothers in employment protection do not have a bias about their possible choice restriction in the next period. They correctly assume they can return to employment, less the probability of a plant closure, during that time. Since the model has a finite horizon, it can be solved by backwards induction using equations (12) and (13) for a given set of parameters.

6 Identification and Structural Estimation

This section develops a revealed preferences approach to recover expectations about future employment opportunities from choice data. It complements the evidence of section 4 that relies exclusively on stated beliefs.[33] To strengthen this approach, the estimation uses administrative data on labor supply choices, and mostly avoids the usages of survey data.[34] First, the critical elements that identify expectations are discussed, followed by a formal proof. Afterwards, the estimation procedure is presented.

6.1 Identification

To identify the structural model, I combine the underlying idea of bunching-related estimators (see, for example, Saez, 2010; Kleven & Waseem, 2013) with the underlying idea of difference-in-differences estimators (see, for example, Card & Krueger, 1994; Imbens & Wooldridge, 2009). The mass of mothers, who return exactly at the end of their employment protection, contains

[33]There is a lasting debate in economics if researchers should restrict themselves to the usage of revealed preferences, for example by relying on choice data, or if they should also consider data on potentially underlying drivers of choices like beliefs (see Gul & Pesendorfer, 2008; Schotter, 2008; Caplin & Schotter, 2008). While this research makes use of both approaches, there are some potential threats to the interpretation of section 4's evidence. First, as discussed by List (2001) and Loomis (2011), willingness-to-pay measurements elicited via non-incentivized questions generally suffer from a severe hypothetical bias. Even though questions regarding expectations are not a willingness-to-pay measurement, they still might suffer from the same underlying issues. These issues are, for example, that answers are strategic or individuals are rather inattentive regarding future events. Second, the framing and the precise wording of the question might influence subjects differently, resulting in a wide variety of possible interpretations of their answers. Third, the concept of probabilities might be challenging for a significant number of subjects, thus, preventing some individuals from answering the question properly (see for example Garfield & Ahlgren, 1988; Tversky & Kahneman, 1973).

[34]Only partnerships and the involuntary job separation rates are estimated using survey data. In theory, these can also be recovered from administrative data.

information about the average job offer expectations of mothers. Exploiting maternity leave reforms that extend the employment protection period permits to isolate the mass that is exclusively related to expectations. As a result, I am able to separately identify real job offer arrival rates, the expectation of these rates, and individual preferences.

6.1.1 Identification Approach

The end of employment protection marks a change in the probability to be able to choose employment in the next period. During the protection, mothers can freely choose between staying non-employed and returning to employment since they are entitled to their previous held position. The only threat to this free choice is the risk of a plant closure. If mothers extend their child-related career breaks beyond the protection period, they have to rely on a job offer to be able to return to employment. The change from an employment guarantee to an uncertain situation at the end of maternity leave creates a trade-off for mothers, who prefer to have extended career breaks. These have to choose between returning to a guaranteed job now, or returning later, but then having to rely on a job offer. Since they have to make this decision at the end of the protection period, their expectations about future job offer rates are crucial. As a result, some mothers return right at the end of the employment protection period, not because they prefer to work, but because they expect a low job offer rate.

Figures 2 and 3 describe this situation graphically. Both figures abstract from working hours and simplify the decision to a binary choice between non-employment (NE) and employment (E). Figures 2 visualizes the underlying choice process of the next period conditioned on the choice in the current period. It differentiates between current choices without employment protections and the current choice of non-employment when in employment protection. Only when an individual chooses non-employment, the expected job offer rate directly affects the expected future value of the subsequent period. The switch from the underlying process at the end of the employment protection is visualized in Figure 3.

The higher a mother expects her job arrival rate to be, the lower she expects to be restricted in her future choices once the employment protection period ends. The lower the average expectations to be restricted, the lower the average likelihood to return at the end of her maternity leave. In the extreme, if a mother expects to always find employment, the end of employment protection does not present a rise in her expected probability to be restricted in future choices. As a result, there is

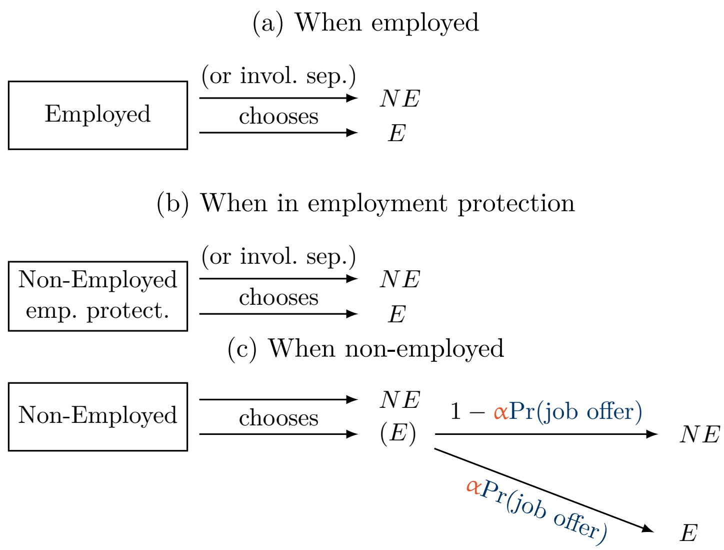

Figure 2: Underlying decision processes

no distinct reason to return right before the end of maternity leave. In contrast, if a mother believes that there are no future employment opportunities, then returning within the maternity leave period presents the only option for her to return to employment. In this case, the probability of a mother returning at the end of the employment protection is rather high. The average individual returning probability at the end of the employment protection is proportional to the number of mothers who return at this point compared to other periods. Thus, the number of mothers returning contains information about the average expectations of the future job offer probability. An important detail of this process is that mothers have to decide if they want to return before the end of the employment protection, and thus, before they can experience the real labor market conditions. They have to rely on their expectations regarding future employment possibilities without being able to rely on any experience.

Figure 4 visualizes the identification of the job offer expectations. The left panel illustrates the situation when time is assumed to be continuous. The right panel illustrates the situation when time is assumed to be discrete. The side-by-side placement provides a better understanding of the translation from the underlying preferences (top left panel) to the observed outcomes in the data (bottom right panel). All x-axes denote the time since the birth of the youngest child. In this illustration, the employment protection is assumed to end with period three. The top graphs plot the difference in the value functions between non-employment and employment, while the bottom

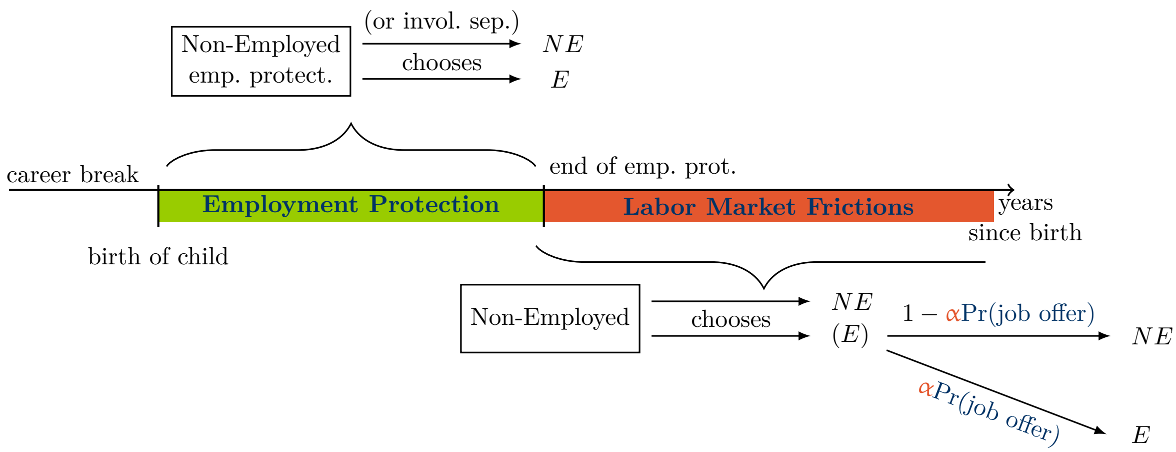

Figure 3: Change in underlying decision process

graphs show the density of mothers returning to employment depending on the age of the youngest child.[35]

Three scenarios are illustrated. The first scenario (dotted black line) represents the counterfactual situation when the employment protection does not end with period 3, but continues forever. It equals the scenario in which individuals expect the job offer rate to be exactly as high as the probability of not losing employment due to a plant closure. The second scenario (solid green line) depicts a situation in which individuals have rational expectations of their future employment possibilities. The third scenario (dashed blue line) represents individuals with upwards biased expectations. These individuals are overestimating the probability of finding employment after the end of the employment protection. However, they anticipate that the probability is lower than in the protection period, and therefore, differ from the individuals of scenario I.

To identify the expected job offer rate, the discontinuity in the value function is crucial. As seen by the three scenarios, the more optimistic individuals are on average, the lower the relative number of returnees at the end of the employment protection. This links the mass of returning mothers to the average future job offer expectations. In the underlying process, the majority of women would return to employment just before the end of the employment protection. In the discrete case, the

[35] All graphs assume that if employment protection never ends, the difference between the value of non-employment and employment shrinks over time. This is mainly due to the decreasing current utility of leisure as the child gets older. Additionally, the more the human capital depreciates while not working, the smaller the difference between the future values of non-employment and employment gets. In the graph, it is assumed that the first effect dominates the second. That the density is not monotonically increasing in the bottom figures is due to the assumption that the majority of individuals are assumed to have returned before the end of period 3.

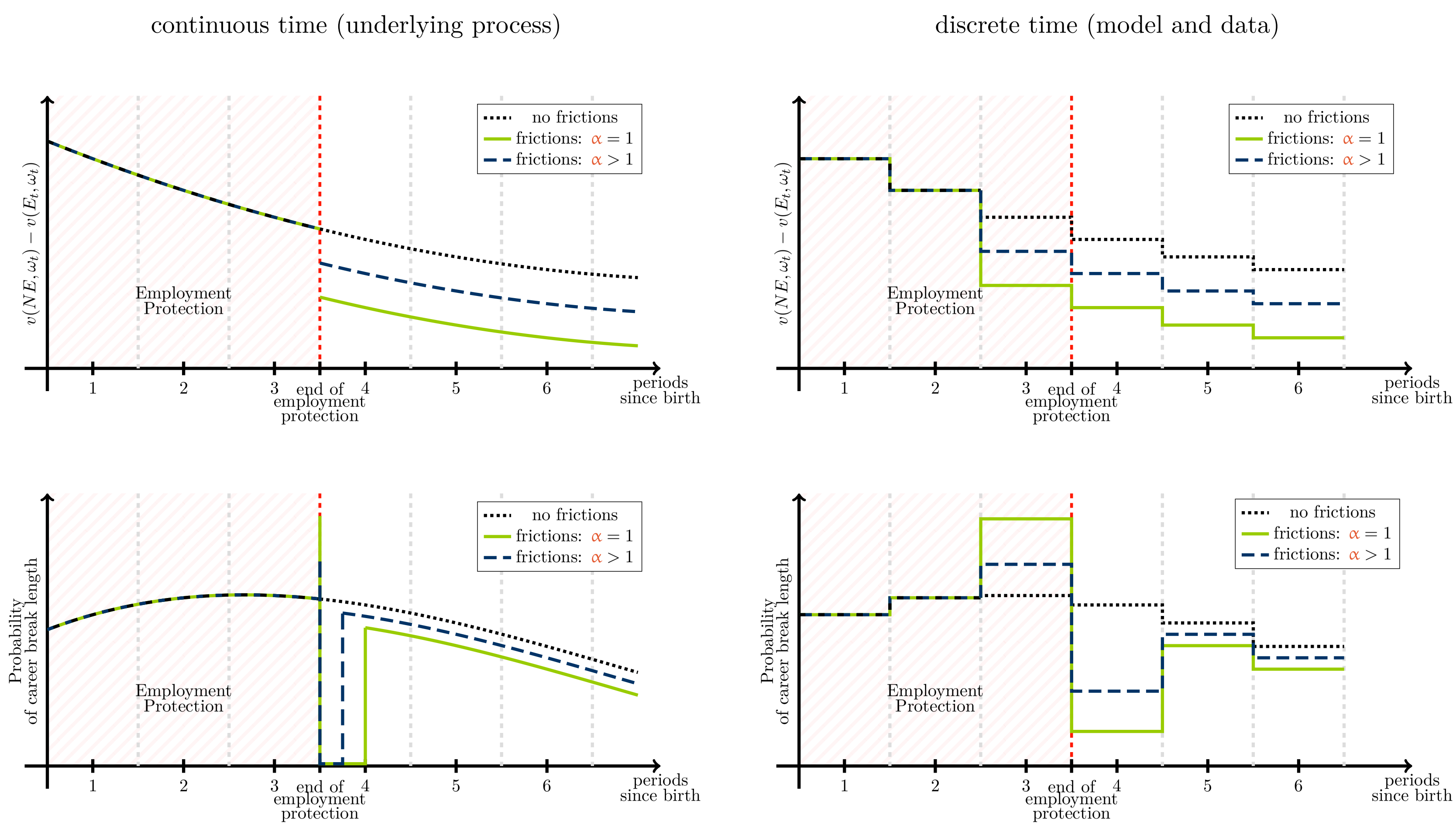

Figure 4: Identification of expectations

Notes: The figure visualizes the identification of the job offer expectations. The left half illustrates the situation when time is assumed to be continuous, the right panel depicts the process when time is assumed to be discrete. All x-axes denote the time since the birth of the youngest child. The employment protection is assumed to end with period three. The top graphs plot the difference in the value functions between non-employment and employment, while the bottom graphs show the density of mothers returning to employment depending on the age of the youngest child.

majority returns in the last period of the employment protection, spanning over more time. It is important that the decision to return within the employment protection is made before actually facing the real labor market frictions. Therefore, the individual can only have expectations of the job offer probability without any actual experience.

Separating Expectations from Preferences and Real Job Offer Rates. In principle, the bunching of returnees at the end of employment protection can be caused by other discontinuities in the value function. For example, the bottom graphs of figure 4 could look similar if leisure preferences discontinuously change at the end of period three. To isolate the excess mass that is exclusively caused by job offer expectations, it is necessary to know the counterfactual scenario in which the employment protection goes beyond the third period. In addition, to identify the real job offer rates, a counterfactual scenario is needed in which the employment protection ends before the third period. The maternity leave regimes discussed in section 2 provide such counterfactual-like scenarios. Figure 5 visualizes the time lines of these regimes. It shows all three regimes and their respective lengths of employment protection depending on the years since childbirth. The green bars on top of a regime's timeline indicate that individuals are in maternity leave, while the red bars below indicate that the individual has to receive a job offer if she desires to re-enter employment.

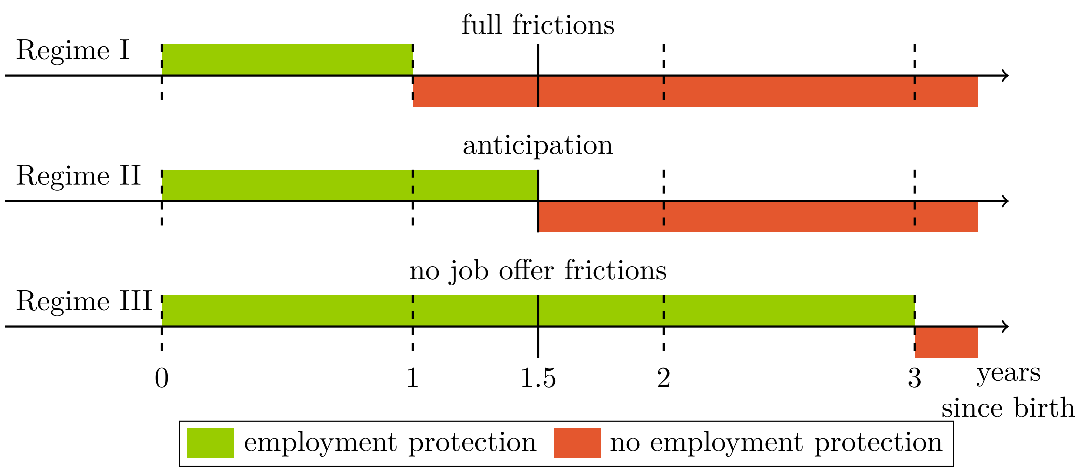

Figure 5: Identification: policy reforms

The different employment protection durations of the three regimes exogenously create three groups. Each group faces a different situation when the youngest child reaches the age of 1.5 years. In regime II, mothers face the end of the employment protection. The number of women returning to employment before the end of period 1.5 is partly driven by job offer expectations. Mothers in regime III, do not face the end of their employment protection when their youngest child reaches the age of 1.5. Thus, the number of mothers who return in regime III before the end of period 1.5 are not influenced by their job offer expectations. These returning mothers can be used to control for discontinuities in preferences at this age. Mothers in regime I do no longer have employment protection. The transition rates of these mothers at can be used to identify the real job offer rate.[36] The idea is that once preferences are identified using transitions from regime III, transitions in regime I provide an indication how restricted mothers are in their choices.

6.1.2 Formal Identification

The formal identification discussion distinguishes between observed choice probabilities and genuine choice probabilities. While restrictions can influence observed choice probabilities (i.e. staying non-employed due to not receiving a job offer), genuine choice probabilities exclusively reflect preferences. Having assumed a type-I extreme value distribution with zero mean, genuine choice probabilities are of the form

$$\mathrm{GPr}\left(C \in CS|\, \omega_t\right) = \frac{\exp\left(v_t\left(C, \omega_t\right)\right)}{\sum\limits_{j \in CS} \exp\left(v_t\left(j, \omega_t\right)\right)}, \tag{14}$$

where C denotes the labor supply choice and CS the individual's choice set. As is formally shown in appendix B.1, all genuine choice and job offer probabilities can be recovered from the data given the structure of the model discussed in section 5. For a given observed state space, the identification relies on a system of six independent equations to identify six unknown probabilities. Besides using observed transition rates, the proof requires the independent identification of the job separation rate.[37] The identification result holds for periods for which individuals enjoy employment protection and for periods for which they do not. Having recovered all genuine choice and true job offer probabilities allows for the recovery of the parameters of the utility function, the job offer probabilities and the wage process. This is formally shown in appendix B.2.1.

To reduce the space of some equations, the following notation is introduced:

$$\mathrm{LS}(\{CS\}|\omega_{t+1}) = \ln\left(\sum_{j \in CS} \exp(v_{t+1}(j, \omega_{t+1}))\right). \tag{15}$$

[36] Appendix B shows that the real job offer rates can be recovered even without relying on the different regimes.
[37] I identify the job separation rate by using additional survey data.

Identification of Expected Job Offer Probabilities. The formal identification of the job offer expectations builds on the differences in genuine choice probabilities between the two maternity leave regimes I and II. After the employment protection in regime II ends, individuals in both regimes face the same future value functions conditioned on their previous choices. This equality permits to cancel out future terms. As a result, the proof does not require strong functional form assumptions. In particular, the identification does not rely on the functional form of the utility function, the wage process or the discount factor β.[38] The crucial element for the identification of expectations is the end of the employment protection and its resulting discontinuity in future expected values. To shorten notation, the state space is often abbreviated by only the critical variables. All other variables are assumed to be equal between all groups. The starting point is the log ratio of two genuine choice probabilities for no-longer-fertile[39] women having a child of age one:

$$\begin{aligned}
&\mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = I\right) = \ln\left(\frac{\mathrm{GPr}\left(PT \in \{NE, PT, FT\} \middle| ac_t = 1, jp_t = 1, r_t = I\right)}{\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| ac_t = 1, jp_t = 1, r_t = I\right)}\right) = \\
&= u\left(PT, ac_t = 1\right) - u\left(NE, ac_t = 1\right) \\
&\quad + \beta \sum_{\omega_{t+1} \in \Omega_{t+1}} \Big\{ \delta v_{t+1}\left(NE, \omega_{t+1}\right) + (1-\delta)\left(1 - \alpha^{FT}\pi^{FT}\right) \mathrm{LS}\left(\{NE, PT\} \middle| \omega_{t+1}\right) \\
&\qquad\qquad + (1-\delta)\, \alpha^{FT}\pi^{FT} \mathrm{LS}\left(\{NE, PT, FT\} \middle| \omega_{t+1}\right) \Big\} q\left(\omega_{t+1}, jp_{t+1} = 0 \middle| PT, r_t = I\right) \\
&\quad - \beta \sum_{\omega_{t+1} \in \Omega_{t+1}} \Big\{ \left(1 - \alpha^{PT}\pi^{PT}\right)\left(1 - \alpha^{FT}\pi^{FT}\right) v_{t+1}\left(NE, \omega_{t+1}\right) \\
&\qquad\qquad + \alpha^{PT}\pi^{PT}\left(1 - \alpha^{FT}\pi^{FT}\right) \mathrm{LS}\left(\{NE, PT\} \middle| \omega_{t+1}\right) \\
&\qquad\qquad + \left(1 - \alpha^{PT}\pi^{PT}\right) \alpha^{FT}\pi^{FT} \mathrm{LS}\left(\{NE, FT\} \middle| \omega_{t+1}\right) \\
&\qquad\qquad + \alpha^{PT}\pi^{PT}\alpha^{FT}\pi^{FT} \mathrm{LS}\left(\{NE, PT, FT\} \middle| \omega_{t+1}\right) \Big\} q\left(\omega_{t+1}, jp_{t+1} = 0 \middle| NE, r_t = I\right),
\end{aligned} \tag{16}$$

where $\mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = I\right)$ is introduced to simplify notation by denoting the logarithm of the ratio of the genuine choice probabilities. Rearranging and plugging in the definitions

[38] The proof requires that the researcher knows the distribution of the flow utility errors. As it is standard for these models, I assume that these errors are extreme value type-I with zero mean distributed.

[39] Assuming that there are no future children simplifies the formal discussion, but is not necessary for identification.

of the genuine choice probabilities leads to

$$
\begin{aligned}
&\mathrm{LR}\left(PT, NE \middle| ac_t=1, jp_t=1, r_t=I\right) = u\left(PT, ac_t=1\right) - u\left(NE, ac_t=1\right) \\
&\quad + \beta \sum_{\substack{\omega_{t+1} \\ \in \Omega_{t+1}}} \Big\{ v_{t+1}\left(NE, \omega_{t+1}\right) - (1-\delta)\left(1-\alpha^{FT}\pi^{FT}\right) \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT\} \middle| \omega_{t+1}\right)\right) \\
&\qquad - (1-\delta)\,\alpha^{FT}\pi^{FT} \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right) \Big\} q\left(\omega_{t+1}, jp_{t+1}=0 \middle| PT, r_t=I\right) \\
&\quad - \beta \sum_{\substack{\omega_{t+1} \\ \in \Omega_{t+1}}} \Big\{ v_{t+1}\left(NE, \omega_{t+1}\right) - \alpha^{PT}\pi^{PT}\left(1-\alpha^{FT}\pi^{FT}\right) \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT\} \middle| \omega_{t+1}\right)\right) \\
&\qquad - \left(1-\alpha^{PT}\pi^{PT}\right)\alpha^{FT}\pi^{FT} \ln\left(\mathrm{GPr}\left(NE \in \{NE, FT\} \middle| \omega_{t+1}\right)\right) \\
&\qquad - \alpha^{PT}\pi^{PT}\alpha^{FT}\pi^{FT} \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right) \Big\} q\left(\omega_{t+1}, jp_{t+1}=0 \middle| NE, r_t=I\right).
\end{aligned}
\tag{17}
$$

Since these women are in the last period of employment protection, staying non-employed in the current period forces them to rely on future job offers if they desire to re-enter employment later. In contrast, women in regime II can remain non-employed for one additional period, before they have to rely on future job offers to re-enter employment. Their difference in the expected future value functions is

$$
\begin{aligned}
&\mathrm{LR}\left(PT, NE \middle| ac_t=1, jp_t=1, r_t=II\right) = u\left(PT, ac_t=1\right) - u\left(NE, ac_t=1\right) \\
&\quad + \beta \sum_{\substack{\omega_{t+1} \\ \in \Omega_{t+1}}} \Big\{ v_{t+1}\left(NE, \omega_{t+1}\right) - (1-\delta)\left(1-\alpha^{FT}\pi^{FT}\right) \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT\} \middle| \omega_{t+1}\right)\right) \\
&\qquad - (1-\delta)\,\alpha^{FT}\pi^{FT} \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right) \Big\} q\left(\omega_{t+1}, jp_{t+1}=0 \middle| PT, r_t=II\right) \\
&\quad - \beta \sum_{\substack{\omega_{t+1} \\ \in \Omega_{t+1}}} \Big\{ v_{t+1}\left(NE, \omega_{t+1}\right) \\
&\qquad - (1-\delta) \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right) \Big\} q\left(\omega_{t+1}, jp_{t+1}=1 \middle| NE, r_t=II\right).
\end{aligned}
\tag{18}
$$

By subtracting (18) from (17), it is possible to eliminate instantaneous utilities and the future value functions of part-time employment. Since transition probabilities are equal for both regimes, the remaining terms can be summarized with one large sum:[40]

$$
\begin{aligned}
&\mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = I\right) - \mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = II\right) = \\
&= \beta \sum_{\omega_{t+1} \in \Omega_{t+1}} \Big\{ \alpha^{PT} \pi^{PT} \left(1 - \alpha^{FT} \pi^{FT}\right) \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT\} \middle| \omega_{t+1}\right)\right) \\
&\qquad + \left(1 - \alpha^{PT} \pi^{PT}\right) \alpha^{FT} \pi^{FT} \ln\left(\mathrm{GPr}\left(NE \in \{NE, FT\} \middle| \omega_{t+1}\right)\right) \\
&\qquad + \alpha^{PT} \pi^{PT} \alpha^{FT} \pi^{FT} \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right) \\
&\qquad - (1 - \delta) \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right) \Big\} q\left(\omega_{t+1} \middle| NE, \omega_t\right).
\end{aligned}
\tag{19}
$$

Besides α^{PT} and α^{FT}, all elements in equation (19) are identified. Note that the equation can be derived for all state spaces ω_t, for instances for different ages, working histories, or partnership states. As a result, there are as many identifying equations as there are different state spaces in Ω_{t+1}, as long as these affect genuine choice probabilities. With α^{PT} and α^{FT} appearing in linear form, both parameters are identified from two different state spaces $\omega_t \neq \omega_t'$. Appendix B.3 proofs this by solving equation (19) for both parameters respectively. Assuming that individuals are not myopic, it is possible to divide various versions of the equation by each other, such that the identification does not depend on the value of $\beta \neq 0$. This is formally shown in appendix B.3. Further, note that equation (19) does not rely on the functional form assumptions or the particular parameters of the utility function or the wage process.

The equation also reflects the intuitive arguments made in section 6.1.1. If individuals are underestimating the job arrival rate to a maximum extend ($\alpha^{PT} = \alpha^{FT} = 0$), that is they expect to never receive a job offer again, equation (19) reduces to

$$
\begin{aligned}
&\mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = I\right) - \mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = II\right) = \\
&\quad = \beta \sum_{\omega_{t+1} \in \Omega_{t+1}} \left\{ -(1 - \delta) \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right) \right\} q\left(\omega_{t+1} \middle| NE, \omega_t\right),
\end{aligned}
\tag{20}
$$

which is positive, since the logarithm of a probability is negative. As appendix B.4 formally shows, the derivative of equation (19) with respect to either α^{PT} or α^{FT} is negative. Thus, in line with

[40]It is assumed that there are no financial differences between being non-employed with employment protection and being non-employed without employment protection. This simplifies the formal identification argumentation. Since maternity benefits are means tested in both regimes, this refers to groups that do not qualify for these benefits. Additional variation exploitable for identification comes also from individuals receiving maternity benefits that differ from benefits received in non-employment. This is possible since the structural model and its estimation can account for differences in incomes, once consumption preferences are identified.

the intuitive arguments of section 6.1.1, the higher the expected job arrival rate, the smaller the difference between the two regimes. This difference even turns negative for individuals that expect to always find a job ($\alpha^{PT} = \frac{1}{\pi^{PT}}$ and $\alpha^{FT} = \frac{1}{\pi^{FT}}$). In this case, equation (19) reduces to

$$\begin{aligned} &\mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = I\right) - \mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = II\right) = \\ &= \beta \sum_{\omega_{t+1} \in \Omega_{t+1}} \left\{\delta \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right)\right\} q\left(\omega_{t+1} \middle| NE, \omega_t\right), \end{aligned} \tag{21}$$

which is negative, since the involuntary job separation rate δ is rather small. Under the expectation to always receive a job offer, having a job has a lower future expected value than being non-employed. This is due to the fact that if one has a job, there is a small probability to loose such job and as a result has to stay non-employed for the period. In contrast, being non-employed and expecting to always receive a job offer does not lead to a choice restriction.

6.2 Estimation Procedure

The estimation procedure is divided into two parts. In a first step, the discount factor is set, and the exogenous parameters and processes are estimated. In a second step, the parameters of the structural model are estimated. The semi-annual discount factor β is set to $\sqrt{0.98}$, the square root of the annual discount factor found, for example, in Attanasio et al. (2008) and Blundell et al. (2016). Both studies employ similar utility functions, which allow for non-separability of leisure and consumption in the framework of female labor supply over the life-cycle.[41] Job separations are estimated via a linear regression quadratic in age and occur with an average probability of 4.8%, decreasing over the life-cycle. Childcare costs are estimated as averages for children under three (€1.82 per hour) and for children between three and six (€1.15 per hour). The exogenous processes of marrying and divorcing partners, of the partner's income, and of the arrival of children are estimated with the method of simulated, relying on averages over the life-cycle. Appendix C provides further details on these estimations.

In the second step, relying on the parameters estimated in the first step, a method of simulated moments estimation is carried out to recover the structural parameters of the model.[42] To estimate the 42 parameters of the model, a total of 490 moments are used. These consist mostly of conditional

[41] Haan & Prowse (2017) use the same discount factor in an estimation based on SOEP data.

[42] Due to computational limitations, the second step does not incorporate that parameters of the first step are estimated.

choice probabilities and transition rates. To identify the job offer expectations, regime specific employment choices are targeted. An overview of the distribution of moments is provided in table 5.

Table 5: Overview of moments

Moments	Number	Structural parameters primarily identified
(1)	(2)	(3)
Average full-time and part-time employment rates, unconditional, and conditional on partnerships, presence of a child, age of the youngest child	21	Utility function parameters
Transition rates from non-employment to employment, from employment to non-employment, and from employment to employment	30	Job offer probabilities
life-cycle employment rates, overall, full-time and part-time	192	Utility function parameters and job offer probabilities
Part-time and full-time employment rates depending on the age of the youngest child	111	Leisure preferences depending on the age of the youngest child
Log wages at beginning of the life-cycle, log wage distribution for full-time and part-time, change of wages after non-employment spell	19	Human capital and wage process parameters
Log wages of the life-cycle	64	Human capital and wage process parameters
Regime specific employment rates depending on the age of the youngest child	48	Job offer expectations

Given the set of moments, parameters are estimated by the method of simulated moments. This method tries to maximize the similarity between the simulated data and the observed data, where similarity refers to the chosen moments. The estimation procedure is as follows:

1. For a given set of parameters (Θ), the described model is solved via backwards induction.

2. Given the choice-specific value functions, all life-cycle decisions for all observed women are simulated. For each woman in the sample, ten life-cycles are simulated.

3. For a given woman, all periods that are not observed in the SOEP data are deleted in the simulated data.[43] Steps 1, 2 and 3 result in a simulated data set with ten times as many observations as in the observed SOEP data.

4. For the simulated and the observed sample all moments are computed and the value of the following objective function is computed:

$$f(\Theta) = \left\{ \sum_{k=1}^{K} \left[\left(M_k^d - \frac{1}{s} \sum_{s=1}^{10} M_k^s(\Theta) \right)^2 / \ \mathbb{V}\mathrm{ar} \left(M_k^d \right) \right] \right\} \tag{22}$$

   where K is the number of moments, M_k^d denotes the k-th data moment, and M_k^s the k-th simulated data moment using data from replication s.

5. Given the value of the objective function, the optimization algorithm then chooses new parameters.

6. Steps 1 - 5 are repeated until $\hat{\Theta} = \arg\min_{\Theta} f(\Theta)$ is found.

Note that equation (22) does not use the asymptotically optimal weighting matrix, because of its poor small sample properties (see Altonji & Segal, 1996). Instead, I use a diagonal matrix with sample variances of the respective moments as its elements. These variances are estimated using bootstrapping with clustering at the individual level.[44] Since the simulated choices are discrete outcomes, the objective function is a step function and does not possess valid derivatives at all points. Therefore, a pattern search method is employed, which is a derivative-free optimization routine. It is implemented here using the Dakota toolkit (see Adams et al., 2013) that allows for parallelization. Standard errors of Θ are estimated following Gourieroux et al. (1993).

[43] Furthermore, wages are only recorded when the simulated individual is employed and the original SOEP interview took place at the given period. To account for non-random missing wages, a linear probability model is used to fit the probability of not observing a wage given the state space variables. Simulated wages are then deleted according to this probability.

[44] I use 1001 replications following Davidson & MacKinnon (2000).

7 Empirical Results

7.1 Goodness of fit

Figures 6, 7, and 8 illustrate the fit of the model. While the observed data is presented in solid blue lines, the estimated model is shown in dashed magenta lines. The model nicely reflects the employment behavior over the life-cycle. While part-time work mostly increases over the life-cycle, full-time work decreases until the age of 35. Although the utility function does not include age related terms, the model can reproduce these trends. The model has some issues fitting the very last periods, which might be related to the model not including a retirement decision, but rather ending at age 50. Thus, toward the end of the model, dynamics introduced by human capital accumulation are slowly eliminated and choices only depends on the instantaneous utility-leisure trade-off. In real life, individuals might continue working, to accumulate more capital towards retirement.

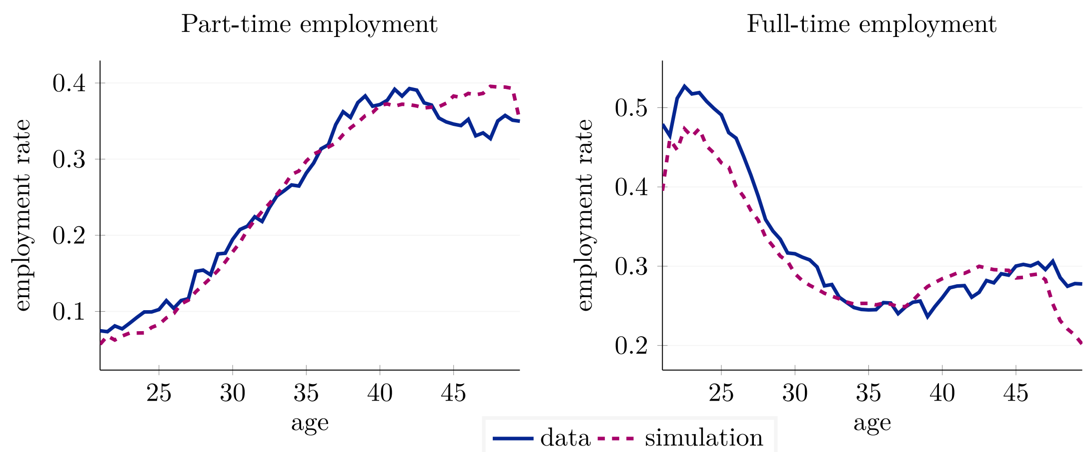

Figure 6: Employment rates by age

Notes: Comparison of the observed and simulated female employment rates over the life-cycle. Observed rates are in solid blue lines and are based on the SOEP data. Simulated rates are in dashed magenta lines and are based on the estimated model.

An important part for the estimation of career costs caused by child related employment interruptions is a close fit of employment behavior around childbirth. As figure 7 illustrates, the model can replicate these choices accurately. Before having a child, women are equally likely to be employed in the data and in the simulations. This is important, since only employed individuals have the right to return to employment during maternity leave. In particular, the model closely fits the first three years after the birth of a child. These are the most important moments for identification of the job offer beliefs. Overall, the model can replicate the larger return to part-time employment within the first five years, while also fitting the overall trend in full-time employment.

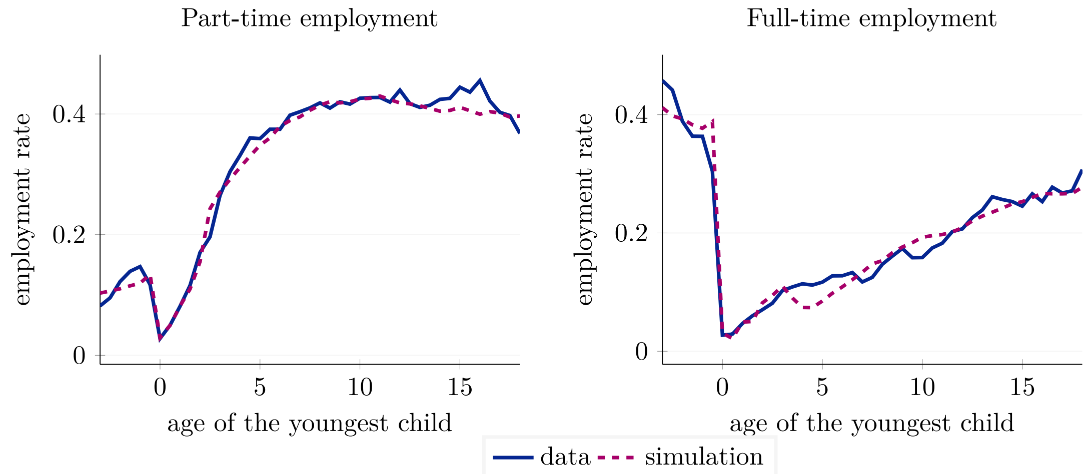

Figure 7: Employment rates by time to/since childbirth

Notes: Comparison of the observed and simulated female employment rates by time to/since childbirth. Observed rates based on SOEP data are in solid blue lines. Simulated rates estimated based on the estimated model are in dashed magenta lines.

In addition to the employment rates, the model is able to replicate average wages over the life-cycle. The average wage increase at the beginning of the working life until the age of 30 is especially well fitted. Afterwards, wages in both the simulation and the data stagnate. At the end of the life-cycle, average wages only slightly increase, which is replicated by the model. Overall, the good fit of wages insure that the costs of biased beliefs can be accurately estimated.

7.2 Parameter Estimates

Before discussing the parameters of the job offer probabilities and their expectations, it is helpful to look at some of the estimated parameters of the utility function and wage process that are reported in tables 6 and 7, respectively. The risk preference parameters γ_c is slightly higher than values usually found in the literature,[45] indicating a higher risk aversion of the sampled women compared to other studies. Because the parameters is greater than one, the utility function becomes negative for all values, thus the higher a parameter in the exponential, the lower the total level of utility.

[45] For example, Haan & Prowse (2014) estimate a CRRA risk aversion parameter of about 2.54 in a study regarding labor supply and retirement decisions using SOEP data.

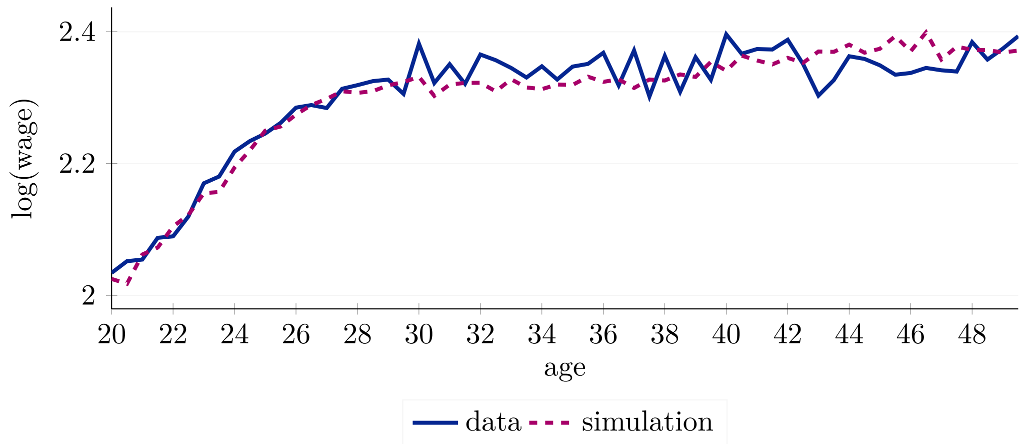

Figure 8: Mean log wage rates over the life-cycle

Notes: Comparison of the observed and simulated log wages over the life-cycle. Observed log wages are in solid lines. Log wages simulated based on the estimated model are in dashed magenta lines.

Considering the values of the estimated parameters, $\gamma_{part\text{–}time}$ is smaller than $\gamma_{full\text{–}time}$, indicating that having less leisure time, in general, decreases utility. The age of the youngest child has a different effect on utility from part-time and full-time leisure. The estimated parameters for leisure preferences, depending on the age of the youngest child, imply that as the child gets older, especially after the age of ten, utility from part-time leisure slightly decreases. In contrast, for utility derived from working full-time increases for almost all ages of the youngest child. It is also notable that the parameters for the presence of a partner and persistence are both small and insignificant for part-time, but large and significant for full-time work.

Table 7 reports the estimated parameters of the wage function and the human capital process. Individuals without any on-the job human capital receive average wages of €6.69.[46] An additional year of full-time employment increases wages by 21.2% at the beginning of the working life, while after working full-time for ten years, an additional year only results in a wage increase of about 1.8%. Human capital depreciates with a yearly rate of 4.7%, which is important when regarding part-time employment. Human capital accumulation for part-time workers corresponds to only 5.7% of the full-time accumulation. Working part-time can only increase wages for individuals with a very low human capital stock, while for other groups, human capital accumulation in part-time work can barely compensate for depreciation and, thus, might even lead to losses in human capital. Blundell et al. (2016) term this effect in combination with the persistence in working choices, the

[46]The value is for 2000 and corresponds to $10.50 in 2018.

Table 6: Estimates of preference parameters

	Part-Time Employment		Full-Time Employment	
	Coeff.	St. Error	Coeff.	St. Error
	(1)	(2)	(3)	(4)
General ($\gamma_{\cdot}$)	0.306296	(0.012680)	0.457867	(0.025300)
Partner ($\gamma_{\cdot,\,partner}$)	0.002645	(0.058486)	−0.947004	(0.025304)
Y. child's age, cons. ($\gamma_{\cdot,\,ac0}$)	0.041199	(0.058486)	−0.581309	(0.012682)
Y. child's age, lin. ($\gamma_{\cdot,\,ac1}$)	−0.022504	(0.000376)	0.021216	(0.000414)
Y. child's age, qu. ($\gamma_{\cdot,\,ac2}$)	0.003229	(0.000075)	−0.003966	(0.000117)
State dependency	0.029302	(0.058473)	0.981887	(0.012682)
Risk preferences (γ_c)	2.997577 (0.051999)			

Notes: The table reports estimated preference parameters. The first row reports the overall taste for part-time and full-time employment, the second row the taste for part-time and full-time when a partner is present. Rows three, four and five report the quadratic modeled preferences for both employment states depending on the age of the youngest child present. Row six reports the added utility when choosing the same employment state as last period and row seven the CRRA parameter.

part-time penalty. In general, the estimated parameters are close to their findings, although for Germany and not the United Kingdom.[47]

In addition to parameters for the real offer rate for part- and full-time employment, the parameter determining expectations (α) is reported in table 8. For an easier interpretation of these results, the yearly real and expected offer rates are depicted in figure 9. Without any potential non-employment time (ρ), individuals receive offers for part-time employment with a probability of 17.1% and for full-time employment with a probability of 50.8% in a given year.[48]. While the probability of receiving a part-time offer only decreases slightly over time, the probability of full-time employment strongly decreases with more potential years spent non-employed. With ten years of potential non-employment, the yearly offer rates drop to 9.0% for part-time employment and 19.9% for full-time employment.

Agents have strongly biased beliefs regarding their opportunity to be able to return to em-

[47] Blundell et al. (2016) find slightly lower values for $\gamma_{wage,\,e}$ of 0.152 for women with secondary education and 0.229 for women with high school education. For η, their yearly values translate to half-yearly values of 0.0414 (secondary education) and 0.0289 (high school education) and, respectively, for λ of 0.0766 and 0.0487.

[48] Since the decision period in the model is semi-annual, the yearly job offer rates are computed as follows: $\pi^{yearly} = 1 - (1 - \pi^{half-yearly})^2$

Table 7: Wage and human capital parameters

	Coeff. (1)	St. Error (2)
Intercept ($\gamma_{wage,\, const.}$)	6.6924	(0.007703)
Returns to experience ($\gamma_{wage,\, e}$)	0.2786	(0.003056)
Depreciation rate (η)	0.0240	(0.000961)
Human capital accum. while in part-time (λ)	0.0285	(0.001635)
Variance wage shock (σ_{xi})	0.2498	(0.002937)

Table 8: Employment offers

	Part-Time Employment		Full-Time Employment	
	Coeff. (1)	St. Error (2)	Coeff. (3)	St. Error (4)
$\gamma_{JO\cdot,c}$	−2.318044	(0.013012)	−0.855202	(0.001692)
$\gamma_{JO\cdot,\rho_1}$	−0.084939	(0.000336)	−0.087805	(0.001036)
$\gamma_{JO\cdot,\rho_2}$	0.001395	(0.000059)	−0.004080	(0.000058)
α		1.657566	(0.013012)	

ployment, with an overestimate of roughly 66%. Given the small standard error, a null-hypothesis that individuals have rational expectations can be rejected at all common significance levels. The estimated parameter also aligns perfectly with the evidence presented in section 4. If the two-year values of table 4 are transformed to half-yearly values, the average α is about 1.31. However, as mentioned earlier, this value might be driven by individuals who are long-term unemployed and have lost faith in finding new employment. For groups with higher expectations of future employment opportunities, the half-yearly α lies between 1.67 and 2.43.[49] The estimate from the structural model aligns greatly with the sample after excluding the most pessimistic individuals.

[49]For the group stating a probability greater than or equal to 30% for finding employment within two years, the half-yearly α equals 1.67, for the group stating the probability greater than or equal to 50%, it is 1.79, and for the group stating the probability greater than or equal to 8 0% it is 2.43.

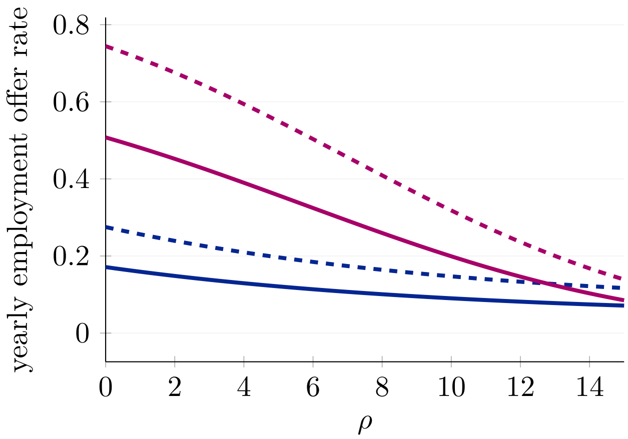

Figure 9: Employment offers and expectations

Notes: Employment offers varying with potential non-employment time ρ. Offer for part-time employment are in blue, while offers for full-time employment are in magenta. Real offer rates are in solid lines, expected rates are in dashed lines.

7.3 Overconfidence vs. rational Expectations

Given the estimated parameters, it is possible to quantify the costs of biased expectations. To do so, I randomly draw $10,000$ individuals from my sample and simulate their life-cycle decisions, once with biased expectations and once with rational expectations, holding everything else constant. Table 9 reports the differences between biased and rational expectations regarding child-related career breaks. In regime I, overconfidence causes these breaks to last 5.8 months longer, an increase of 11.2%. In regime II, breaks are 7.8 months longer, corresponding to an increase of 17.6%, and in regime III, biased beliefs result in breaks lasting 4.7 months longer, a rise of 13.9%. All these numbers include mothers who entered employment at some point after having a child. The results of rows four and five of table 9 show that biased beliefs also affect behavior with respect to entering employment at all after having a child. For regime I, over 20.4% fewer women enter employment after having a child when having biased beliefs. Due to the higher percentage of working mothers in the other two regimes, their numbers are even higher. In regime II, 40.6% more mothers never return to employment after having a child when overestimating future employment opportunities, while in regime III, this increase is about 29%.

To quantify the costs of biased beliefs, I compute the net present values of earnings, income and consumption, from the birth of the first child through the end of the model. Table 10 provides

Table 9: Overconfidence and child related career breaks

	Regime I (1)	Regime II (2)	Regime III (3)
Avg. career break ($\alpha = 1.66$)	4.8004	4.3241	3.1776
Avg. career break ($\alpha = 1$)	4.3137	3.6776	2.7900
Share of mothers, not enter. emp. ($\alpha = 1.66$)	15.66%	13.19%	7.64%
Share of mothers, not enter. emp. ($\alpha = 1$)	13.00%	9.38%	5.92%

Notes: Results are based on 10,000 simulated life-cycles, once with the estimated beliefs of job offer expectations ($\alpha = 1.66$) and once with rational expectations ($\alpha = 1$). Average career breaks are in years.

an overview of these costs depending on the respective regime. Column (1) refers strictly to labor market earnings and does not include any benefits. Depending on the regime, life-cycle earnings decrease between 18% and 12%, when overestimating job offer probabilities. From a public economics point of view, these results are striking, since lower labor market earnings correspond to lower taxable income. However, costs are lower for individuals since maternity leave, unemployment, and child benefits mitigate some of the lost earnings. As column (2) of table 10 shows, individual income only drops by 4.13% for regime I, 7.39% for regime II, and 5.06% for regime III. These numbers are still considerably large, compared to overall career costs of children of 35% found by Adda et al. (2017). Accommodating for the potential partners' income only slightly reduces the career costs. As stated before, husbands are assumed to work full-time and do not interrupt their working careers due to the birth of a child. Since most women do work part-time, their contribution to the overall household earnings is also not particularly high under rational expectations, thus limiting the reduction in consumption when women have biased beliefs.

8 Conclusion

The birth of a child strongly impacts the working careers of women, especially since a majority of mothers remain at home for an extended period of time before re-entering employment. The length of such career breaks is influenced by the expectations of future employment possibilities. Overestimating these possibilities might cause mothers to not return within the maternity leave pe-

Table 10: Costs of biased expectations

	Earnings (1)	Income (2)	Consumption (3)
Regime I	−16.36%	−4.13%	−3.39%
Regime II	−18.28%	−7.39%	−4.14%
Regime III	−12.41%	−5.06%	−3.07%

Notes: Results are based on 10,000 simulated life-cycles, once with the estimated beliefs of job offer expectations ($\alpha = 1.66$) and once with rational expectations ($\alpha = 1$). Earnings refers to direct labor market earnings. Income additionally includes maternity leave, unemployment and child benefits. Consumption includes income and additionally income of the partner, but are weighted by household members like in the utility function. Values correspond to net present values from the birth of the first child until the end of the model.

riod during which their employment is protected, although non-optimal with rational expectations. Thus, upwards biased beliefs cause longer career interruptions, a higher fraction of mothers never returning to employment, and higher career costs of children.

I develop a structural life-cycle model of female labor supply and human capital accumulation, allowing for non-rational expectations of future job arrival rates. To identify expectations within the model, I derive a novel identification strategy that allows recovering the key parameters from observed labor supply choices. The strategy exploits a discontinuity in the future expected value of non-employment caused by the end of employment protection. In combination with maternity leave reforms that change the duration of the employment protection after the birth of a child, it is possible to separately identify expectations, job-arrival rates, and preferences. I estimate the model using survey data from the German Socio-Economic Panel Study, since the German setting provides the necessary variation for identification.

Indeed, estimations show that mothers highly overestimate their chances on the labor market, since they expect the half-yearly job arrival rate to be 66% higher than the actual rate, on average. Comparing simulations with the estimated preference parameters – one restricted to rational expectations and one with the estimated expectations – shows that overconfidence prolongs career breaks between 4.7 and 7.8 months on average, depending on the length of employment protection. This results in a reduction of lifetime earnings from employment between 12% and 18%. Some of these losses are mitigated by various benefits and earnings of a potential husband, thus actual consumption losses only range from 3.1% to 4.1%.

The results have important implications from a public economics perspective. In addition to increasing social security spending, prolonged career breaks cause lower earnings from employment that translate directly into forgone tax revenue. The consequences for the individual are also substantial. The income loss causes reduced pension benefits, thus, contributing to an increased risk of poverty in retirement. Since biased expectations can be interpreted as market failures and because of their far-reaching consequences, interventions by policy makers might be justified. Possible policies could aim to provide better information about labor market conditions, for example by sending official information letters to new families, or financial incentives to return within the period the individual's job is protected.

Overall, some caution is appropriate when interpreting these results, since the model only estimates an average bias for all individuals and does not model an explicit retirement decision. Adding heterogeneity in expectations might be valuable, since individuals most likely exhibit different degrees of overconfidence causing different magnitudes of career costs. While heterogeneity based on observables can be estimated using the presented strategy, including unobservable heterogeneity demands a strong refinement of the identification approach. Extending the model by including a retirement decision presumably results in an increase in the costs of overconfidence, since the lost earnings from employment result in a lower average pension income. However, it is not immediately evident if individuals will postpone their retirement to overcome these losses. Future work might incorporate these elements into the model and, to simulate the effects of possible policies, aim to reduce the cost of overconfidence. These can be, for example, an increase in in-work benefits conditioned on returning within the employment protection or further prolonging employment protection without prolonging maternity benefits.

Appendix A: Suggestive Evidence: Additional Information

A.1 SOEP Questions

The suggestive evidence of section 4 is based on the following two SOEP questions. Only individuals who respond (b), (c), or (d) in the first question are asked the second question.

Do you intend to engage in paid employment (again) in the future?

(a) No, definitely not

(b) Probably not

(c) Probably

(d) Yes, definitely

The original questionnaire also underlines the words "next two years" at the following question.

How likely is it that one or more of the following occupational changes will take place in your life within the <u>next two years</u>?

	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
Start paid work	0	10	20	30	40	50	60	70	80	90	100
Become self-employed	0	10	20	30	40	50	60	70	80	90	100
Receive further education	0	10	20	30	40	50	60	70	80	90	100

A.2 Registered Unemployment Rate for Germany between 1999 and 2007

A possible explanation for the gap between the stated likelihood to find employment and the realizations is that individuals were affected by a shared macro shock. If the economy falls into an unexpected recession, it is harder for everyone to find employment and, thus, it is natural to expect a gap between stated preferences and realizations. Figure 10 shows that this is unlikely to drive the results of section 4. It plots the unemployment rate for the relevant years of table 4. The questions were first asked in 1999 and then again in 2001, 2003, and 2005. Although the SOEP interview can happen at any time throughout the year, the majority are in spring. The possible interview times are indicated by the shaded pink areas in the figure. The gray area marks a recession according to the definition of the OECD.

Overall the unemployment rate did not fluctuate much, staying mostly around 10 %. In two of the four years the questions were asked, a recession followed, while in the other two years a decrease in the unemployment rate followed. Overall, this should at least partly balance the macro influence on the realizations.

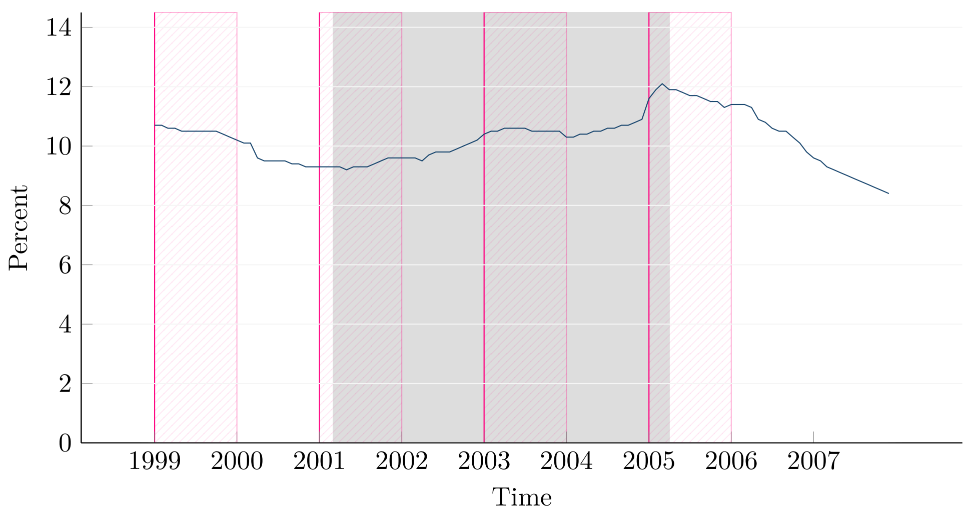

Figure 10: Registered unemployment rate for Germany

Notes: The blue line depicts the monthly registered unemployment rate in Germany. The gray area marks recessions as defined by the OECD. The shaded pink areas indicate the year over which subjects were asked about their employment expectations. Sources: Federal Reserve Bank of St. Louis (2017a), Federal Reserve Bank of St. Louis (2017b).

Appendix B: Proofs of Section 6.1.2

The results derived in this section strongly inform the choice of moments for estimating the model. However, the method of simulated moments uses all moments simultaneously, and thus, the estimation procedure is not sequential as the following discussion might suggest. Moments might also not exactly represent identification equations, since these equations might be functions of various moments.

B.1 Identification of Genuine Choice Probabilities and Job Offer Rates

The proof starts with the identification discussion when individuals are not in employment protection. With three possible choices, there are nine observable transition probabilities for a given state space ω_t,[50]. Six of them are independent, since choice probabilities have to add up to one conditioned on last period's employment state. Observed choice probabilities are denoted by $\mathrm{OPr}\left(C_t | \, C_{t-1}, \omega_t\right)$, where $C_t \in \{NE, PT, FT\}$ denotes the observed labor supply choice (non-employed, part-time employed, full-time employed). Similar to the observed choice probabilities, there are seven genuine choice probabilities. Four of them are independent. These probabilities are denoted by $\mathrm{GPr}\left(C_t \in CS_t | \, \omega_t\right)$, where CS_t denotes the choice set from which the economic agent can choose. In addition to these four genuine choice probabilities, there are two unknown job offer probabilities for a given state space. Thus, six independent equations identify six unknowns, a sufficient condition.[51]

[50]Note that not all values of the state space are directly observed. To control for human capital, it is possible to compare groups of individuals with the exact same working history.

[51]In addition to the six independent equations, there are two inequality conditions for each unknown ensuring that probabilities are not smaller than zero and not larger than one.

Proof. With a known job separation rate $\delta(\omega_t)$,[52] one set of six independent equations is

$$\begin{aligned} \mathrm{OPr}\left(PT_t|\, NE_{t-1},\omega_t\right) =& \pi^{PT}\left(\omega_t\right)\Big[\Big(1-\pi^{FT}\left(\omega_t\right)\Big)\mathrm{GPr}\left(PT\in\{NE,PT\}|\,\omega_t\right) \\ & + \pi^{FT}\left(\omega_t\right)\mathrm{GPr}\left(PT\in\{NE,PT,FT\}|\,\omega_t\right)\Big], \end{aligned} \tag{23}$$

$$\begin{aligned} \mathrm{OPr}\left(FT_t|\, NE_{t-1},\omega_t\right) =& \pi^{FT}\left(\omega_t\right)\Big[\Big(1-\pi^{PT}\left(\omega_t\right)\Big)\mathrm{GPr}\left(FT\in\{NE,FT\}|\,\omega_t\right) \\ & + \pi^{PT}\left(\omega_t\right)\mathrm{GPr}\left(FT\in\{NE,PT,FT\}|\,\omega_t\right)\Big], \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{\mathrm{OPr}\left(PT_t|\, PT_{t-1},\omega_t\right)}{1-\delta\left(\omega_t\right)} =& \Big(1-\pi^{FT}\left(\omega_t\right)\Big)\mathrm{GPr}\left(PT\in\{NE,PT\}|\,\omega_t\right) \\ & + \pi^{FT}\left(\omega_t\right)\mathrm{GPr}\left(PT\in\{NE,PT,FT\}|\,\omega_t\right), \end{aligned} \tag{25}$$

$$\frac{\mathrm{OPr}\left(FT_t|\, PT_{t-1},\omega_t\right)}{1-\delta\left(\omega_t\right)} = \pi^{FT}\left(\omega_t\right)\mathrm{GPr}\left(FT\in\{NE,PT,FT\}|\,\omega_t\right), \tag{26}$$

$$\frac{\mathrm{OPr}\left(PT_t|\, FT_{t-1},\omega_t\right)}{1-\delta\left(\omega_t\right)} = \pi^{PT}\left(\omega_t\right)\mathrm{GPr}\left(PT\in\{NE,PT,FT\}|\,\omega_t\right), \tag{27}$$

$$\begin{aligned} \frac{\mathrm{OPr}\left(FT_t|\, FT_{t-1},\omega_t\right)}{1-\delta\left(\omega_t\right)} =& \Big(1-\pi^{PT}\left(\omega_t\right)\Big)\mathrm{GPr}\left(FT\in\{NE,FT\}|\,\omega_t\right) \\ & + \pi^{PT}\left(\omega_t\right)\mathrm{GPr}\left(FT\in\{NE,PT,FT\}|\,\omega_t\right). \end{aligned} \tag{28}$$

To ease notation, the state space is dropped henceforth. The job offer rate for part-time employment can be identified from dividing (23) by (25):

$$\pi^{PT} = \frac{\mathrm{OPr}\left(PT_t|\, NE_{t-1}\right)(1-\delta)}{\mathrm{OPr}\left(PT_t|\, PT_{t-1},\right)}. \tag{29}$$

Similarly, the full-time job offer rate can be recovered by dividing (24) by (28):

$$\pi^{FT} = \frac{\mathrm{OPr}\left(FT_t|\, NE_{t-1},\right)(1-\delta)}{\mathrm{OPr}\left(FT_t|\, FT_{t-1},\right)}. \tag{30}$$

Using equations (29) and (30), the genuine choice probabilities for choosing either part-time or

[52] I identify the job separation rate by using additional survey data.

full-time employment from the general choice set can be identified by equations (26) and (27) respectively:

$$\begin{aligned}\mathrm{GPr}\left(FT \in \{NE, PT, FT\}\right) &= \frac{\mathrm{OPr}\left(\left. FT_t \right| PT_{t-1}\right)}{(1-\delta)\,\pi^{FT}} \\ &= \frac{\mathrm{OPr}\left(\left. FT_t \right| PT_{t-1}\right) \mathrm{OPr}\left(\left. FT_t \right| FT_{t-1}\right)}{(1-\delta)^2\, \mathrm{OPr}\left(\left. FT_t \right| NE_{t-1}\right)},\end{aligned} \tag{31}$$

$$\begin{aligned}\mathrm{GPr}\left(PT \in \{NE, PT, FT\}\right) &= \frac{\mathrm{OPr}\left(\left. PT_t \right| FT_{t-1}\right)}{(1-\delta)\,\pi^{PT}} \\ &= \frac{\mathrm{OPr}\left(\left. PT_t \right| FT_{t-1}\right) \mathrm{OPr}\left(\left. PT_t \right| PT_{t-1}\right)}{(1-\delta)^2\, \mathrm{OPr}\left(\left. PT_t \right| NE_{t-1}\right)}.\end{aligned} \tag{32}$$

The genuine probability of choosing part-time employment when restricted to a choice between non-employment and part-time employment is recovered by

$$\begin{aligned}&\mathrm{GPr}\left(PT \in \{NE, PT\}\right) = \frac{\mathrm{OPr}\left(\left. PT_t \right| PT_{t-1}\right)}{(1-\delta)\left(1-\pi^{FT}\right)} - \frac{\pi^{FT}}{1-\pi^{FT}} \mathrm{GPr}\left(PT \in \{NE, PT, FT\}\right) \\ &= \frac{\mathrm{OPr}\left(\left. PT_t \right| PT_{t-1}\right)\left[\mathrm{OPr}\left(\left. PT_t \right| NE_{t-1}\right) \mathrm{OPr}\left(\left. FT_t \right| FT_{t-1}\right) - \mathrm{OPr}\left(\left. FT_t \right| NE_{t-1}\right) \mathrm{OPr}\left(\left. PT_t \right| FT_{t-1}\right)\right]}{(1-\delta)\, \mathrm{OPr}\left(\left. PT_t \right| NE_{t-1}\right)\left[\mathrm{OPr}\left(\left. FT_t \right| FT_{t-1}\right) - (1-\delta)\, \mathrm{OPr}\left(\left. FT_t \right| NE_{t-1}\right)\right]}.\end{aligned} \tag{33}$$

Similarly, the genuine choice probability of choosing full-time employment when restricted to a choice between non-employment and full-time employment is identified by

$$\begin{aligned}&\mathrm{GPr}\left(FT \in \{NE, FT\}\right) = \frac{\mathrm{OPr}\left(\left. FT_t \right| FT_{t-1}\right)}{(1-\delta)\left(1-\pi^{PT}\right)} - \frac{\pi^{PT}}{1-\pi^{PT}} \mathrm{GPr}\left(FT \in \{NE, PT, FT\}\right) \\ &= \frac{\mathrm{OPr}\left(\left. FT_t \right| FT_{t-1}\right)\left[\mathrm{OPr}\left(\left. FT_t \right| NE_{t-1}\right) \mathrm{OPr}\left(\left. PT_t \right| PT_{t-1}\right) - \mathrm{OPr}\left(\left. PT_t \right| NE_{t-1}\right) \mathrm{OPr}\left(\left. FT_t \right| PT_{t-1}\right)\right]}{(1-\delta)\, \mathrm{OPr}\left(\left. FT_t \right| NE_{t-1}\right)\left[\mathrm{OPr}\left(\left. PT_t \right| PT_{t-1}\right) - (1-\delta)\, \mathrm{OPr}\left(\left. PT_t \right| NE_{t-1}\right)\right]}.\end{aligned} \tag{34}$$

Having recovered these probabilities, the last three genuine choice probabilities can be computed as follows

$$\mathrm{GPr}\left(NE \in \{NE, PT\}\right) = 1 - \mathrm{GPr}\left(PT \in \{NE, PT\}\right), \tag{35}$$

$$\mathrm{GPr}\left(NE \in \{NE, FT\}\right) = 1 - \mathrm{GPr}\left(FT \in \{NE, FT\}\right), \tag{36}$$

$$\begin{aligned}\mathrm{GPr}\left(NE \in \{NE, PT, FT\}\right) = 1 &- \mathrm{GPr}\left(PT \in \{NE, PT, FT\}\right) \\ &- \mathrm{GPr}\left(FT \in \{NE, PT, FT\}\right).\end{aligned} \tag{37}$$

The identification discussion is much briefer for individuals in employment protection. Note that only currently non-working individuals enjoy employment protection. The system of equations reduces to the two following independent equations

$$\frac{\mathrm{OPr}\left(PT_t|\, NE_{t-1}\right)}{(1-\delta)} = \mathrm{GPr}\left(PT \in \{NE, PT, FT\}\right), \tag{38}$$

$$\frac{\mathrm{OPr}\left(FT_t|\, NE_{t-1}\right)}{(1-\delta)} = \mathrm{GPr}\left(FT \in \{NE, PT, FT\}\right). \tag{39}$$

Given the job destruction rate, these two equations directly identify the genuine choice probabilities for part- and full-time employment when being able to choose from all options. ■

B.2 Identification of Model Parameters besides Expectations

B.2.1 Utility Parameters

Proof. Given the assumption that utility shocks are type-I extreme value with zero mean distributed, the parameters can be identified by a combination of the genuine choice probabilities. Central is the logarithm of the ratio of two genuine choice probabilities, for example part-time and non-employment:

$$\begin{aligned}\ln\left[\frac{\mathrm{GPr}\left(PT \in \{NE, PT\}|\, \omega_t\right)}{\mathrm{GPr}\left(NE \in \{NE, PT\}|\, \omega_t\right)}\right] &= u\left(PT, \omega_t\right) + \mathbb{E}\max\left(PT, \omega_t\right) \\ &\quad - u\left(NE, \omega_t\right) - \mathbb{E}\max\left(NE, \omega_t\right)\end{aligned} \tag{40}$$

As only differences in utility can be identified, the utility of non-employment is normalized to be zero. In the last period, the expected maximum of future utilities drops out. This permits the direct identification of the utility of part-time:

$$\ln\left[\frac{\mathrm{GPr}\left(PT \in \{NE, PT\}|\, \omega_T\right)}{\mathrm{GPr}\left(NE \in \{NE, PT\}|\, \omega_T\right)}\right] = \mathrm{LR}\left(PT, NE, \omega_T\right) = u\left(PT, \omega_T\right), \tag{41}$$

where $\mathrm{LR}\left(PT, NE, \omega_t\right)$ denotes the logarithm of the ratio of the two genuine choice probabilities, given state space ω_t.

Risk Aversion. For the identification of γ_c, two groups of individuals, which only differ in their experience, and thus, their wages are compared. The difference in wages results in different consumption opportunities.[53] Denote the state space with high consumption as ω_T^H and the one with low consumption as ω_T^L. Both groups of individuals have neither a partners, nor children. Their utility of working part-time is then given by

$$\frac{\text{LR}\left(PT, NE, \omega_T^H\right)}{\text{LR}\left(PT, NE, \omega_T^L\right)} = \frac{u\left(PT, \omega_T^H\right)}{u\left(PT, \omega_T^L\right)} = \left(\frac{c^H}{c^L}\right)^{1-\gamma_c}. \tag{42}$$

When choosing experience differences such that $\frac{\text{LR}\left(PT,NE,\omega_T^L\right)}{\text{LR}\left(PT,NE,\omega_T^H\right)} > 0$, γ_c is identified by

$$\gamma_c = 1 - \frac{\ln\left(\frac{\text{LR}\left(PT,NE,\omega_T^H\right)}{\text{LR}\left(PT,NE,\omega_T^L\right)}\right)}{\ln\left(c^H\right) - \ln\left(c^L\right)}. \tag{43}$$

Leisure Preferences without Children. Once the risk-aversion parameter is identified, it is straight forward to identify the parameters for working part-time or full-time. Again, consider a group of individuals neither having a partner, nor children:

$$\text{LR}\left(PT, NE, \omega_T\right) = \frac{(c/\bar{c})^{1-\gamma_c}}{1-\gamma_c}\exp\left(\gamma_{PT}\right) \quad \Rightarrow \quad \gamma_{PT} = \ln\left(\text{LR}\left(PT, NE, \omega_T\right)\frac{1-\gamma_c}{(c/\bar{c})^{1-\gamma_c}}\right). \tag{44}$$

Note that γ_c is assumed to be larger than one, thus, the logarithm should exists. In a similar fashion, the parameters γ_{FT}, $\gamma_{P,PT}$ and $\gamma_{P,FT}$ can be identified.

Leisure Preferences with Children. Since the model ends at age 50, it is possible to observe children at various ages in the last period. This allows to recover parameters γ_{PT,ac_0}, γ_{PT,ac_1}, γ_{PT,ac_2}, γ_{FT,ac_0}, γ_{FT,ac_1}, and γ_{FT,ac_2}. Consider three groups of mothers without partners who only differ with respect to the age of their youngest child in the last period. For instances, children with the ages 16, 17, and 18. Denote the state space for these women by $\omega_{T,ac=16}$, $\omega_{T,ac=17}$, $\omega_{T,ac=18}$,

[53] Note that experience exclusively affects utility through consumption.

respectively. The utilities of part-time employment in the last period is then given by

$$\mathrm{LR}\left(PT, NE, \omega_{T,ac=16}\right) = \frac{\left(c/\bar{c}\right)^{1-\gamma_c}}{1-\gamma_c}\left[\gamma_{PT,ac_0} + \gamma_{PT,ac_1} \times 16 + \gamma_{PT,ac_2} \times 16^2\right], \tag{45}$$

$$\mathrm{LR}\left(PT, NE, \omega_{T,ac=17}\right) = \frac{\left(c/\bar{c}\right)^{1-\gamma_c}}{1-\gamma_c}\left[\gamma_{PT,ac_0} + \gamma_{PT,ac_1} \times 17 + \gamma_{PT,ac_2} \times 17^2\right], \tag{46}$$

$$\mathrm{LR}\left(PT, NE, \omega_{T,ac=18}\right) = \frac{\left(c/\bar{c}\right)^{1-\gamma_c}}{1-\gamma_c}\left[\gamma_{PT,ac_0} + \gamma_{PT,ac_1} \times 18 + \gamma_{PT,ac_2} \times 18^2\right]. \tag{47}$$

These three linearly independent equations are sufficient to identify the three unknown parameters. Similarly, it is possible to identify the respective parameters for full-time employment.

The leisure parameters for particular ages of the youngest child cannot be recovered from the last period. This is due to the circumstance that children of these ages are not observed in the last period. To identify these parameters, it is necessary to solve the model by backwards induction using the already identified parameters. The model has to be solved backwards until the period for which the first unknown parameter for leisure preferences with respect to a child's age is reached. Denote the age of the youngest child by $\breve{ac}$. The state space for a woman with no partner, but a child with this particular age is denoted by $\omega_{t,\breve{ac}}$. The logarithm of the genuine choice probabilities can again be used for identification:

$$\begin{aligned}\mathrm{LR}\left(PT, NE, \omega_{t,\breve{ac}}\right) =& \mathbb{E}\max\left(PT, \omega_{t,\breve{ac}}\right) - u\left(NE, \omega_{t,\breve{ac}}\right) - \mathbb{E}\max\left(NE, \omega_{t,\breve{ac}}\right) \\ &+ \frac{\left(c_t/\bar{c}\right)^{1-\gamma_c}}{1-\gamma_c}\left[\gamma_{PT,ac_0} + \gamma_{PT,ac_1} \times \breve{ac} + \gamma_{PT,ac_2} \times \breve{ac}^2 + \gamma_{PT,\breve{ac}}\right].\end{aligned} \tag{48}$$

Again, $u(NE, \cdot)$ is normalized to be zero. Both $\mathbb{E}\max(\cdot)$ terms are identified by the previously recovered parameters and the construction of the expected maximum, provided that α is identified. The only unknown parameter in equation (48) is $\gamma_{PT,\breve{ac}}$ and thus, can be recovered from the equation. The parameter for full-time is identified by an analogous equation. With parameters $\gamma_{PT,\breve{ac}}$ and $\gamma_{FT,\breve{ac}}$, the model can be solved backwards by one more period. From this period, the parameters $\gamma_{PT,\breve{ac}-0.5}$ and $\gamma_{FT,\breve{ac}-0.5}$ can be recovered. With this method, all other parameters depending on the age of the youngest child can be identified.

B.2.2 Job Offer Parameters

With $\pi^{PT}(\omega_t)$ and $\pi^{FT}(\omega_t)$ being recovered from the data, the identification of the according parameters is straightforward. Consider the following equation:

$$\pi^{PT}(\omega_t) = \frac{\exp\left(LF\left(\gamma_{JO,PT}, \omega_t\right)\right)}{1 + \exp\left(LF\left(\gamma_{JO,PT}, \omega_t\right)\right)}, \tag{49}$$

where $LF(\cdot)$ is a linear function of the respective parameters and the state space. Equation (49) can be transformed to

$$\ln\left[\frac{\pi^{PT}(\omega_t)}{1 - \pi^{PT}(\omega_t)}\right] = LF\left(\gamma_{JO,PT}, \omega_t\right). \tag{50}$$

As long as $LF(\cdot)$ is a linear function, the values of the respective parameters can be recovered by an appropriate system of equations of the corresponding state space values.

B.2.3 Wage Parameters

Since employment experience strongly influences wages, the first working period is key for the identification. Let w_0 denote the wage in the period in which individuals do not have any employment experience. The log-wage in this first working period can be written as

$$\ln(w_0) = \ln(\gamma_{w,const.}) + \xi_{i,t}. \tag{51}$$

Taking the expectation and rearranging leads to

$$\gamma_{w,const.} = \exp\left(\mathbb{E}\left[\ln(w_0)\right]\right), \tag{52}$$

and thus, identifies $\gamma_{w,const.}$. To recover $\gamma_{w,e}$, the wage of the period after exactly one half-year of full-time employment ($w_{0.5,FT}$) is used:

$$\mathbb{E}\left[\ln(w_{0.5,FT})\right] = \ln(\gamma_{w,const.}) + \gamma_{w,e}\ln(1.5). \tag{53}$$

Rearranging the difference of the wage from before and after the very first period of full-time employment identifies the parameter $\gamma_{w,e}$:

$$\gamma_{w,e} = \frac{\mathbb{E}\left[\ln\left(w_{0.5,FT}\right)\right] - \mathbb{E}\left[\ln\left(w_0\right)\right]}{\ln\left(1.5\right)}. \tag{54}$$

In a similar manner, the part-time addition to on-the-job human capital λ can be recovered. For the identification of λ, the expected wages before and after the very first employment period are used provided the individuals worked part-time:

$$\lambda = 1 - \exp\left(\frac{\mathbb{E}\left[\ln\left(w_{0.5,PT}\right)\right] - \mathbb{E}\left[\ln\left(w_0\right)\right]}{\gamma_{w,e}}\right) = 1 - 1.5^{\left(\frac{\mathbb{E}\left[\ln\left(w_{0.5,PT}\right)\right] - \mathbb{E}\left[\ln\left(w_0\right)\right]}{\mathbb{E}\left[\ln\left(w_{0.5,FT}\right)\right] - \mathbb{E}\left[\ln\left(w_0\right)\right]}\right)} \tag{55}$$

Furthermore, once the parameters $\gamma_{w,const.}$ and $\gamma_{w,e}$ are identified, the depreciation parameter η can be recovered. To do so, the wages before and after an employment break are needed. Consider an individual who is working full-time in t and $t+k+1$, but is non-employed in between. The (potential) wages are then given by

$$\mathbb{E}\left[\ln\left(\omega_t\right)\right] = \ln\left(\gamma_{w,const.}\right) + \gamma_{w,e}\ln\left(e_t + 1\right), \tag{56}$$

$$\mathbb{E}\left[\ln\left(\omega_{t+1}\right)\right] = \ln\left(\gamma_{w,const.}\right) + \gamma_{w,e}\ln\left(\left(1-\eta\right)e_t + 1.5\right), \tag{57}$$

$$\mathbb{E}\left[\ln\left(\omega_{t+k+1}\right)\right] = \ln\left(\gamma_{w,const.}\right) + \gamma_{w,e}\ln\left(\left[\left(1-\eta\right)e_t + 0.5\right]\left(1-\eta\right)^k + 1\right). \tag{58}$$

Although the wage in equation (57) cannot be observed from non-employed individuals, it can be recovered from individuals, who continue to work in period $t+1$. Combining equations (57) and (58) identifies η:

$$\eta = 1 - \left(\frac{\exp\left(\frac{\mathbb{E}[\ln(w_{t+k})|l_t=FT,\ l_{t+s}=NE \text{ with } 0<s<k,\ l_{t+k}\in\{FT,PT\}]-\ln(\gamma_{w,const.})}{\gamma_{w,e}}\right) - 1}{\exp\left(\frac{\mathbb{E}[\ln(w_{t+1})|l_t=FT,\ l_{t+1}\in\{FT,PT\}]-\ln(\gamma_{w,const.})}{\gamma_{w,e}}\right) - 1}\right)^{\frac{1}{k}}. \tag{59}$$

■

B.3 Solving Equation (19) for $\boldsymbol{\alpha}^{PT}$ and $\boldsymbol{\alpha}^{FT}$

To shorten equations, the following notation is used:

$$
\begin{aligned}
\Delta \mathrm{LR}_{\mathrm{I-II}} &= \mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = I\right) - \mathrm{LR}\left(PT, NE \middle| ac_t = 1, jp_t = 1, r_t = II\right), \\
LN_{PT} &= \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT\} \middle| \omega_{t+1}\right)\right), \\
LN_{FT} &= \ln\left(\mathrm{GPr}\left(NE \in \{NE, FT\} \middle| \omega_{t+1}\right)\right), \\
LN_{PT,FT} &= \ln\left(\mathrm{GPr}\left(NE \in \{NE, PT, FT\} \middle| \omega_{t+1}\right)\right).
\end{aligned}
$$

Solving equation (19) for α^{PT} results in

$$
\alpha^{PT} = \frac{\Delta \mathrm{LR}_{\mathrm{I-II}} - \beta \sum\limits_{\omega_{t+1} \in \Omega_{t+1}} \left[\alpha^{FT}\pi^{FT} LN_{FT} - (1-\delta) LN_{PT,FT}\right] q\left(\omega_{t+1} \middle| NE, \omega_t\right)}{\beta \sum\limits_{\omega_{t+1} \in \Omega_{t+1}} \left[\pi^{PT}\left(1 - \alpha^{FT}\pi^{FT}\right) LN_{PT} - \pi^{PT}\alpha^{FT}\pi^{FT}\left(LN_{FT} - LN_{PT,FT}\right)\right] q\left(\omega_{t+1} \middle| NE, \omega_t\right)}. \tag{60}
$$

Solving equation (19) for α^{FT} results in

$$
\alpha^{FT} = \frac{\Delta \mathrm{LR}_{\mathrm{I-II}} - \beta \sum\limits_{\omega_{t+1} \in \Omega_{t+1}} \left[\alpha^{PT}\pi^{PT} LN_{PT} - (1-\delta) LN_{PT,FT}\right] q\left(\omega_{t+1} \middle| NE, \omega_t\right)}{\beta \sum\limits_{\omega_{t+1} \in \Omega_{t+1}} \left[\left(1 - \alpha^{PT}\pi^{PT}\right)\pi^{FT} LN_{FT} - \alpha^{PT}\pi^{PT}\pi^{FT}\left(LN_{PT} - LN_{PT,FT}\right)\right] q\left(\omega_{t+1} \middle| NE, \omega_t\right)}. \tag{61}
$$

To identify α^{PT} without relying on β, two additional abbreviations are introduced:

$$
\begin{aligned}
D\left(\omega_t\right) &= \sum_{\omega_{t+1} \in \Omega_{t+1}} \left[\alpha^{FT}\pi^{FT} LN_{FT} - (1-\delta) LN_{PT,FT}\right] q\left(\omega_{t+1} \middle| NE, \omega_t\right), \\
N\left(\omega_t\right) &= \sum_{\omega_{t+1} \in \Omega_{t+1}} \left[\pi^{PT}\left(1 - \alpha^{FT}\pi^{FT}\right) LN_{PT} - \pi^{PT}\alpha^{FT}\pi^{FT}\left(LN_{FT} - LN_{PT,FT}\right)\right] q\left(\omega_{t+1} \middle| NE, \omega_t\right).
\end{aligned}
$$

Using this notation, α^{PT} can be identified using two state spaces ω_t and $\omega_t' \neq \omega_t$. The respective difference between regime I and II are

$$
\Delta \mathrm{LR}_{\mathrm{I-II}}\left(\omega_t\right) = \beta\left[D\left(\omega_t\right) + \alpha^{PT} N\left(\omega_t\right)\right], \tag{62}
$$

$$
\Delta \mathrm{LR}_{\mathrm{I-II}}\left(\omega_t'\right) = \beta\left[D\left(\omega_t'\right) + \alpha^{PT} N\left(\omega_t'\right)\right]. \tag{63}
$$

Assuming that individuals are not fully myopic, i.e. $\beta \neq 0$, the quotient from (62) and (63) results in

$$\frac{\Delta\mathrm{LR}_{\mathrm{I-II}}\left(\omega_t\right)}{\Delta\mathrm{LR}_{\mathrm{I-II}}\left(\omega_t'\right)} = \frac{\left[D\left(\omega_t\right) + \alpha^{PT} N\left(\omega_t\right)\right]}{\left[D\left(\omega_t'\right) + \alpha^{PT} N\left(\omega_t'\right)\right]}. \tag{64}$$

Rearranging yields

$$\alpha^{PT} = \frac{\Delta\mathrm{LR}_{\mathrm{I-II}}\left(\omega_t'\right) D\left(\omega_t\right) - \Delta\mathrm{LR}_{\mathrm{I-II}}\left(\omega_t\right) D\left(\omega_t'\right)}{\Delta\mathrm{LR}_{\mathrm{I-II}}\left(\omega_t\right) N\left(\omega_t'\right) - \Delta\mathrm{LR}_{\mathrm{I-II}}\left(\omega_t'\right) N\left(\omega_t\right)}. \tag{65}$$

Similarly, an equation for α^{FT} can be derived. Thus, to identify the expectations of part-time and full-time offers without relying on time preferences, it is necessary to derive equation (19) for four different state spaces.

B.4 Derivatives of Equation (19) with respect to α^{PT} and α^{FT}

Using the notation of subsection B.3, the derivative with respect to α^{PT} is given by

$$\frac{\partial\Delta\mathrm{LR}_{\mathrm{I-II}}}{\partial\alpha^{PT}} = \beta \sum_{\substack{\omega_{t+1} \\ \in\Omega_{t+1}}} \Big[\underbrace{\left(1-\alpha^{FT}\pi^{FT}\right)\pi^{PT} LN_{PT}}_{\leq 0} \underbrace{-\pi^{PT}\alpha^{FT}\pi^{FT}\left(LN_{FT} - LN_{PT,FT}\right)}_{\leq 0} \Big] q\left(\omega_{t+1}\left|NE,\omega_t\right.\right). \tag{66}$$

Note that LN_{PT} is lower or equal zero since it is a logarithm of a probability. The term $LN_{FT} - LN_{PT,FT} = \ln\left(1 + \frac{\exp(v_{t+1}(PT,\omega_{t+1}))}{\exp(v_{t+1}(NE,\omega_{t+1}))+\exp(v_{t+1}(FT,\omega_{t+1}))}\right)$ is greater zero. Hence, equation (66) is a sum of negative values and is thus, in total also negative. The same line of argument holds for the derivative with respect to α^{FT}, which is also negative:

$$\frac{\partial\Delta\mathrm{LR}_{\mathrm{I-II}}}{\partial\alpha^{FT}} = \beta \sum_{\substack{\omega_{t+1} \\ \in\Omega_{t+1}}} \Big[\underbrace{\left(1-\alpha^{PT}\pi^{PT}\right)\pi^{FT} LN_{FT}}_{<0} \underbrace{-\alpha^{PT}\pi^{PT}\pi^{FT}\left(LN_{PT} - LN_{PT,FT}\right)}_{\leq 0} \Big] q\left(\omega_{t+1}\left|NE,\omega_t\right.\right). \tag{67}$$

Appendix C: External Processes

C.1 Family Dynamics

The underlying binary processes of marriage, divorce, and the arrival of children are modeled as linear probability models, the parameters of which are estimated via the method of simulated moments using the same optimization algorithm as discussed in subsection 6.2. When simulating

these transitions, predicted probabilities below 0 are reset to 0, and predicted probabilities above 1 are reset to 1. All probabilities are separately estimated for both education groups.
The arrival probability of husbands depends on a three-order polynomial in the woman's age. The divorce probability additionally depends on the presence of children. If there are no children present, the probability only depends on a fourth-order polynomial of the woman's age, while if children are present it additionally depends on a second-order polynomial of the age of the youngest child and an interaction term between the mother's and the youngest child's age.

The probabilities of the arrival of children are separately estimated for the first child, additional children and for the presence of a partner. The models for the first child include a second-order polynomial in woman's age if a partner is present and a fourth-order polynomial in the woman's age if no partner is present. For additional children, further terms are added. In a marriage, an additional birth depends on a fourth-order polynomial of the mother's age and a second-order polynomial of the age of the youngest child. Furthermore, interaction terms of the two ages up to the third-order are added. When there is no partner present, the probability depends on a fifth-order polynomial of the woman's age, a fourth-order polynomials of the youngest child's age and interaction terms of the two ages up to a fourth-order polynomial.[54] Women are not allowed to have a first child after the age of 37.5 and an additional child after 38 corresponding to 99 % of the observed births in the sample.

In addition to figures 11 and 12, table 11 illustrates how the estimated processes (dashed lines) compare to the real data (solid lines). Figure 11 shows how family types vary over the age of the mother. The estimated processes nicely fit marriages and the arrival of children over the whole sample. Similarly well fitted is the age of the youngest child, as well as the age of the youngest child when an additional child arrives. These processes are of special importance for two reasons. First, leisure preferences and, thus, employment rates depend strongly on the age of the youngest child. As long as a mother has an additional child during her maternity leave period, the leave period resets.[55] As table 11 shows, the distribution of the sibling's age when a new child is born is tightly fitted by the simulated data.

Besides affecting women's leisure preferences, husbands add to the household income. Almost all husbands work over the whole lifecycle, which is indicated by an overall sample employment

[54]Since the model's decision period is a half-year, women are not able to have an additional child if the youngest child has not yet reached the age of one.

[55]This is fully integrated into the model.

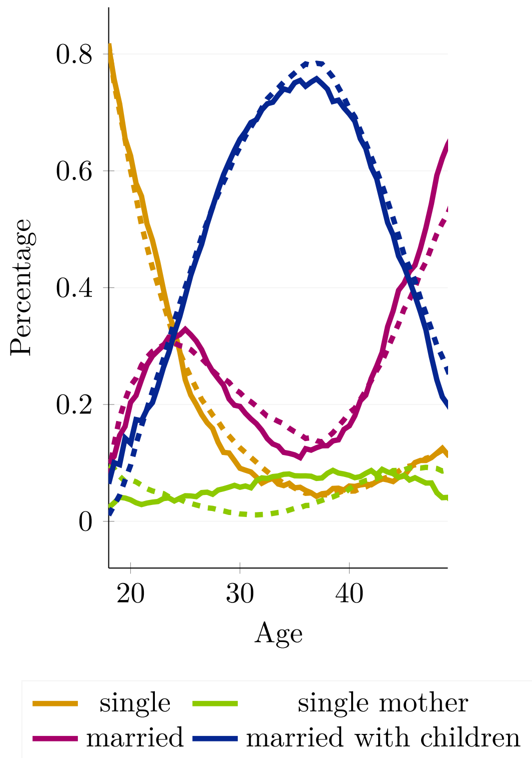

Figure 11: Family dynamics

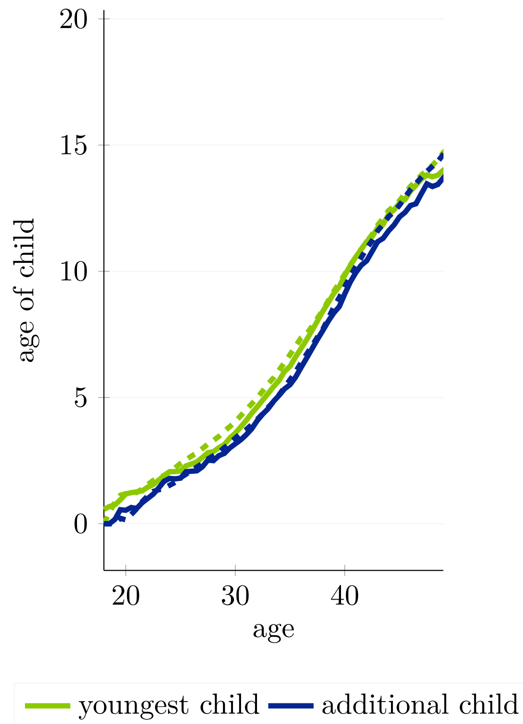

Figure 12: Age of youngest child

Notes: Distribution of family types by age of woman. Data in solid lines, simulations in dashed lines.

rate of 92 % with below 2 % working part-time. To estimate the husband's half-yearly gross income, the observed average income depending on the wife's age are weighted with the respective sample employment rate. These incomes are then used as moments to estimate a linear regression model via the method of simulated moments. Figure 13 illustrates that the simulated values closely fit the observed incomes.

Table 11: Additional child moments

	Low education			
Moment	Data	Sim	SE Data	SE Diff
(1)	(2)	(3)	(4)	(5)
share of add. children	0.5389	0.5442	0.0111	0.4790
age young. sibl. < P10	0.1361	0.1704	0.0131	2.6161
age young. sibl. < P25	0.2571	0.2707	0.0160	0.8447
age young. sibl. < P50	0.5156	0.5119	0.0182	0.2073
age young. sibl. < P75	0.7701	0.7527	0.0157	1.1077
age young. sibl. < P90	0.9007	0.9128	0.0113	1.0808

Notes: Additional moments to estimate family dynamics. Row 1 reports the share of children born when another child is present. Rows 2 - 6 reports the share of additional children born when the youngest sibling's age under the, in the data observed, percentile.

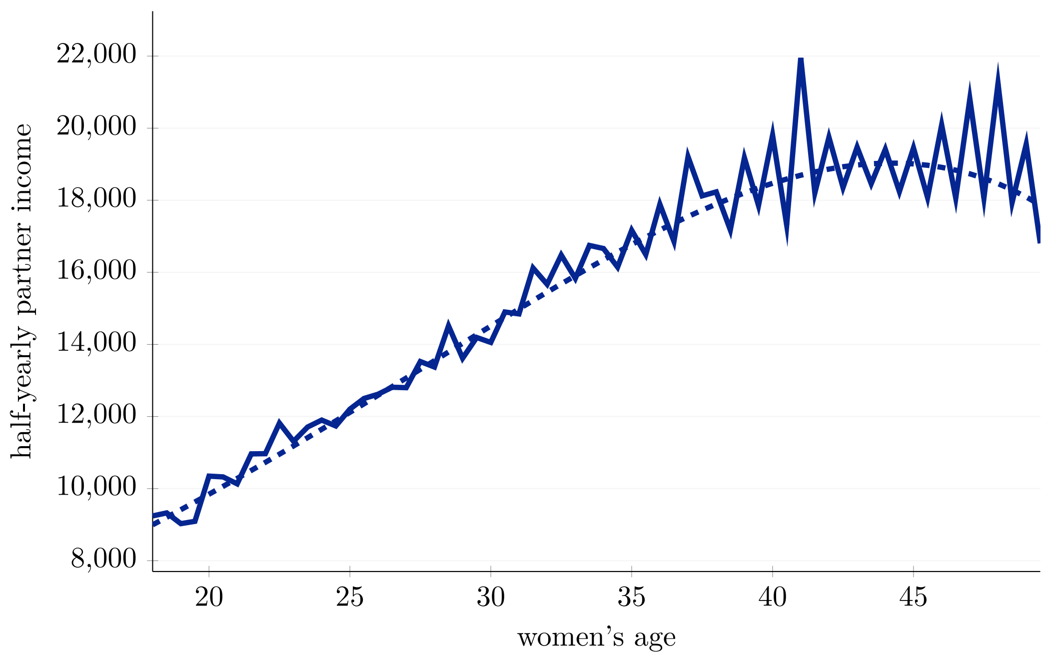

Figure 13: Husband's income

Notes: Potential husband's half-yearly gross income. Data in solid lines, simulations in dashed lines.

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